PROBLEMATISING STATISTICAL MEANINGS: A SOCIOCULTURAL PERSPECTIVE

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We report the findings of a detailed study of the ways in which a group of paediatric nurses think about the notion of average and variation. We describe some continuities and discontinuities between mathematical and nursing epistemologies, and draw some general conclusions about the ways in which more general mathematical meanings are constructed and 'transferred' that takes account of both cognitive and sociocultural perspectives.

INTRODUCTION

Our recent work with a variety of professionals in work settings has convinced us of the superficiality of the widespread view that most professional life has little mathematics in it (e.g. Noss & Hoyles, 1996a; Pozzi, Noss & Hoyles, 1998; Noss, Pozzi & Hoyles, 1999; Noss, Hoyles & Pozzi, 2000; Hoyles, Noss, & Pozzi, 2001; Noss, Hoyles, & Pozzi, in press). There does exist a rich seam of mathematical activity which people exhibit in their working lives, if we know where and when to look, and if we do not rest content to equate reality with the stories people tell about it. So in this work, our starting point has been to study the kinds of problems professionals actually solve, and to try to make visible the basis for their judgements.

Researching in the field of workplace mathematical activity has proved a challenge, as we were wary of taking as a starting point any simple classification of the 'visible mathematics' that *we* could see, but which might merely represent our own, as opposed to our subjects', orientation. Nevertheless, as our research progressed, we became aware that it was possible to make a provisional epistemological classification of some of the activities we were observing, and that it made sense to analyse them in terms of what they represented from our mathematical point of view, as long as this was made explicit. This analysis could then be augmented, synthesised and compared with the practitioners' viewpoints.

In this paper, we report the outcome of one such classification, in which the visible mathematics was the collection, recording and interpretation of data, and, the less visible part, the concepts underpinning the judgements made. The context is a study of paediatric nurses and how they made sense of clinical data, focusing on the concepts of average and variation. The analysis pointed not only to the limitations of previous studies that did not notive invisible mathematics in workplace activity, but also to the subtle and reciprocal mediation of professional and mathematical knowledge. This latter mediation calls into question a purely cognitive approach to investigating statistical understandings, and calls for a recognition of a sociocultural dimension to the problem.

Within an essentially cognitive paradigm, there are numerous findings which appear to illustrate how people's intuitions often seem to stand in the way of developing statistical notions. Shaughnessy (1992) provides a comprehensive overview of this research. With regard to the mean and other measures of central tendency, there appears to be an incompleteness in what people believe, a perceived lack of balance between computational and conceptual understanding, and little understanding of why a mean should be calculated. As far as the understanding of variation is concerned, Shaughnessy notes that: 'people inappropriately believe there is no random variability in the "real world"' (p. 478). This is not only a problem for novices: for example, Batanero *et al* (1994) reported that a widespread misconception amongst university students was a deterministic conception of association; students do not admit exceptions to the existence of a relationship between variables and if they find any reject the idea that there is a relationship.

While providing a wealth of information, these studies can provide only a limited number of starting points for our research, as they fail to acknowledge the role of activity and a range of sense-making devices and strategies that people actually use in practice to describe, for example, randomness and variation. Neither do they identify how knowledge might be re-constructed in the workplace after it is learnt in school. It might be, of course, that there is a simple discontinuity between taught/school and practical mathematical knowledge. Mokros and Russell (1995), in a study of fourth and eighth graders, concluded that some students 'were concerned with procedures, not with meaning. For them, average means a series of steps involving addition and division' (*ibid* p.35). The authors ascribe this procedural strategy to a failure of reification — and it is here that they locate the students' difficulties. There is much in this argument that merits discussion; however, rather than focus on the construction of individual meaning-making, we prefer in this paper to focus on the ways that the professional community of practice shapes the meanings of the statistical ideas.

METHODOLOGY

The practice of nursing involves making sense of a wide range of quantitative information. This clinical experience suggests that nurses might have well-formed intuitions about a wide range of statistical ideas. Moreover, analysis of texts and interviews with nurses suggested that only measurement and recording are regarded as obviously mathematical, while interpretation is mostly seen in terms of clinical knowledge alone. The question for research was this: what meanings do nurses construct as they interpret data within their day-to-day practice?

We gathered data on this issue in three ways: ethnographic observation on the ward, clinical interviews and questionnaires, and a short teaching experiment. We reconstructed our ethnographic data into a set of episodes, each describing a nursing-mathematics activity. While the episodes painted a broad picture of routine practice, we also identified and analysed a number of *breakdown* episodes, which involved a rupture in the normal routines. On these occasions, activities that were normally characterised by unproblematic, routine action were replaced by conflict, disagreement and doubt, which resulted in spontaneous explanations and considerably more articulated (and therefore explicit) reasoning and problem-solving strategies. Although these breakdowns were rare, they provided a rich source of information about participants' views and the knowledge they mobilised in order to substantiate them. Apart from providing insights as they occurred, we reconstructed these episodes and used them as a basis for clinical interviews with a subset of nurses.

THE NURSING-AVERAGE

One breakdown episode observed on the ward involved the interpretation of blood pressure data plotted on a standard nursing chart. We asked the nurses to find the (deliberately vaguely stated) 'average' of a set of blood pressure data plotted on a BP chart (illustrated in Figure 1). Only one nurse used the mean. One mentioned it while adopting another strategy, while reference to either the median or mode was never explicit — although both were used implicitly. The nurses used a variety of strategies, almost all of which would be judged 'wrong' on the straightforward criteria of general mathematical correctness, e.g. *'I looked at the chart and judged which was the middle range'* or *'At a glance, all the systolic pressures are on or around 110 mm Hg'*. But they made perfect sense and were correct as estimates of the child's blood pressure — a kind of everyday average. The graph presented the nurses with a familiar situation in which the *sense* of average was unambiguous, and for which the idea of *representativeness* posed no difficulty because of the naturally symmetrical distribution of hourly BP readings.

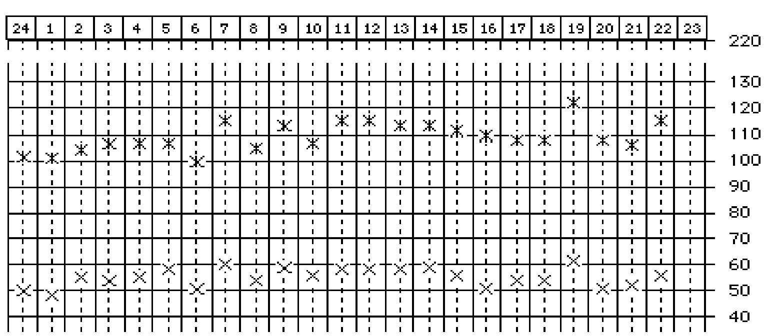

Figure 1. Systolic (top) and diastolic (bottom) blood pressure from a standard patient chart.

In order to make sense of the nurses' strategies, we need to give a more detailed account of the practice underlying them, derived from our ethnographic observations on the wards. When a nurse takes charge of a patient, she immediately establishes his or her *individual baseline*. For example, nurses will attempt to answer questions such as 'what is this patient's normal, stable blood pressure?' This means that nurses have a tendency to take less account of the *whole* body of data when looking at average, and instead focus on clusters of data around the same value — especially if they think the patient was stable or settled around the time of these readings. As all patients have their own individual physiology, their profile of vital signs when they are stable is in some sense unique to them. Establishing a patient's normal profile of vital signs thus enables a nurse to judge when data deviate from the normal. A further feature of this process is that nurses develop an awareness of non-critical factors that lead to unusual readings (e.g. instrument error, patient over-activity) in order to distinguish them from significant changes in the patient's condition.

The notion of an individual baseline is not the only organising idea in nurses' judgements of average, since they must also take account of population norms. Nurses 'know' that blood pressure is related to age, and can usually spontaneously quote a number for the average BP for a given age. But we found that nurses will only use it when it makes sense in terms of clinical practice. For example, the nurses were sceptical of the meaning of an overall population average for a young child and some refused to acknowledge that it existed! Thus the variability of the measures meant there was *no* average — something which makes no sense mathematically but which makes sense for the nurses.

On one level, these are completely understandable responses. The average blood pressure *for an individual child* obviously depends on whether the child was premature, its size and so on. Yet these kinds of considerations are, of course, included in the notion of average — indeed, it is the variability in population data which makes the statistic necessary. As a consequence of their work, nurses have difficulties in considering an average *independently* of the individual because it is the *individual* who is the focus of their care. The fact that 'average' necessarily masks individuality — like any statistic — is therefore seen as problematic. For a nurse, it is the variation that is crucial, and she is prepared to accept the notion of average provided it is individually mediated.

NURSES' CONCEPTS OF VARIATION: BRINGING NURSING MATHEMATICS CLOSER TO MATHEMICS

Following the ethnographic study we designed a teaching experiment for nurses to introduce them to these ideas and we taught our course to 28 paediatric nurses as part of their professional development. We aimed to explore the relationship between what we had identified from our ethnography as nurses' views of average and variation and 'mathematical' views. We used in the course a range of statistical modelling ideas using data analysis software (our primary tool was *Tabletop* [1]). The teaching experiment involved a combination of computer-based hands-on work and whole-class presentations, during which different ways of analysing data were introduced, alongside discussion of the substantive nursing issues involved.

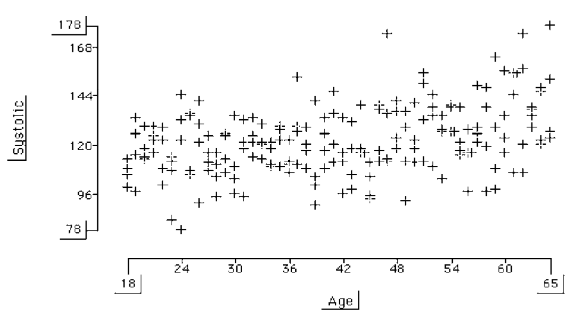

Figure 2. Scattergram of systolic blood pressure and age.

We asked the nurses to consider explicitly the relationship between age and blood pressure, and to write down or illustrate their hypotheses. While most quickly opted to present the data as a scattergram of blood pressure against age, their reaction to this graph (see Figure 2) was as surprising as it was puzzling. Given their practical and theoretical knowledge regarding the relationship between BP and age, we believed that the group would be prone to a high degree of confirmation bias, i.e. finding a relationship of some description regardless of how it was represented on the computer. Instead, many found it difficult to see any relationship at all in the scattergram — and here was the surprise — decided there could be none after all.

It seemed clear that the nurses' judgements were influenced by three main factors. Firstly, the high variation in the blood pressure data was a hindrance to seeing any relationship and obscured the identification of any trend; i.e. variation and relationship were seen as somehow antithetical. Secondly, the rise in the data, in as much as one could be seen, struck many as involving a slope too gentle to indicate a relationship that was so important that it stood as an *a priori* assumption of nursing knowledge, i.e. important relationships should have impressive-looking gradients. Thirdly, the nurses *could* explain the variation on the basis of their practical knowledge — i.e. they were able to reiterate substantive reasons for differences in blood pressure for particular age-groups (lifestyle, health and other physiological factors). Some of the extreme data points were of particular interest, not from the point of view of variation from a trend but because they indicated to the nurses that the data were unreliable. As one nurse put it, 'this patient should be dead!'.

These three factors together — in part predictable from the literature, in part specific to nursing — meant that the nurses tended to focus on variation at the expense of the relationship. At this point, most of the nurses had reached an impasse, so we prompted them to model the data using the other tools available. Many split the data into age groups, finding the mean blood pressure for each group — a familiar number to all the nurses (see Figure 3). This introduction of central tendency allowed many nurses, to our surprise, to reaffirm what they believed but had previously rejected: i.e. that BP did indeed increase with age! The use of the average enabled them to push the variation in the data into the background, as they took on board the increments in the means from the youngest age group to the oldest.

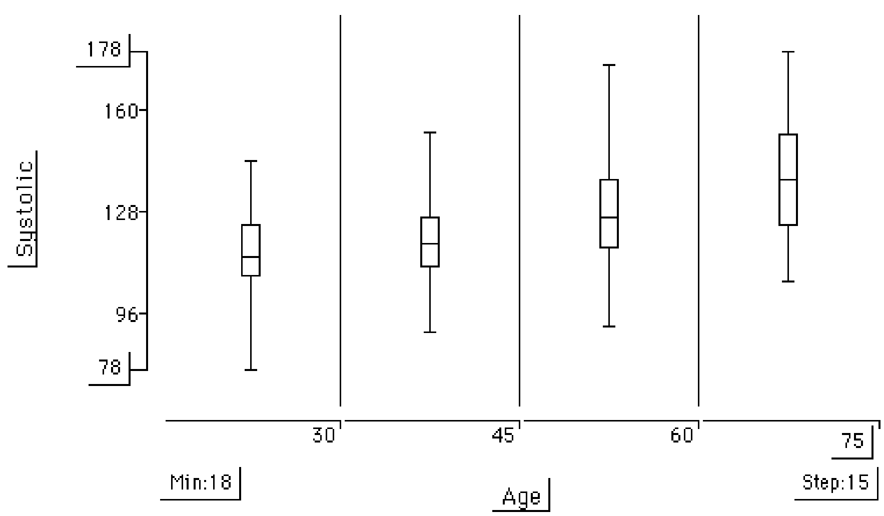

Figure 3. Box-and-whisker plots of systolic pressure grouped by age.

What was it that changed in the nurses' thinking that allowed them to reach this reconciliation? Of course the tools mediated the ways in which the data were conceived — to divide the data into quartiles, to calculate the means — clearly played significant roles. This tool-mediation served as an essential aid in clarifying how the nurses' views developed, and just how their practical knowledge of the average BP of an age group could be effectively mobilised.

DISCUSSION

We begin with a straightforward assertion: no description of the nurses' view of average and variation is adequately characterised in terms of misconceptions of the official, mathematical definitions. The nurses' meanings were different from the mathematician's, interwoven with meanings from their practice, and efficient and effective at work. Nor does it make sense to describe the nurses' strategies as, for example, failing to have reified the notion of average. On the contrary, if reification means anything, it means that an individual has made sufficiently strong links between elements of a concept to ensure its functionality: and as we have seen, this certainly applied to the nurses' notion of average.

What, then, is the status of the nurses' knowledge? The notion of nursing average was, as we have seen, on the one hand a description of the normal state of a patient over time, and on the other, a data point for a population — a measure of location unrelated to variability. In relation to the first aspect, the average provides a baseline — for example, an individual's blood pressure when it is stable. Here the nursing average works like a mathematical average so it can be seen as another example of what we have termed a situated abstraction (see Hoyles, Noss, & Pozzi, 2001; and for a more elaborated discussion, Noss & Hoyles, 1996b). It is abstracted *within* rather than *away from* the nursing setting. It retains crucial elements of the setting in the way it is conceptualised which are not 'noise' from which the mathematical relationship must be extracted; rather it serves as the glue to bind the different elements together. As with a mathematical average, the nursing average is indeed representative of the data as a whole and is recognised as such by nurses, because of the small variability in the data and the approximately symmetrical distribution. Additionally, in this context, outliers really must be explained — as physical 'extraneous variables' or functions of a medical condition.

The population view of an average differs considerably. Contrasting data on an individual with the population from which the individual is drawn is a complex issue from a mathematical perspective. Yet our research showed that the readings taken by nurses, or data which are interpreted by them, take account of population norms in ways that are finely tuned to nursing practice.

When nurses are acting in their familiar situations of monitoring blood pressure over time, they coordinate these different meanings of average to produce functionality. They use the notion of average to reason about their patients' condition, to relate the state of their patient to others in a virtual and deliberately ill-defined population (e.g. an overweight 7 year-old), to communicate information to each other and to doctors, and to make rounded judgements concerning what action to take. Thus, significant background factors in blood pressure readings (e.g. age, sex, obesity, smoking) have to be taken into account by nurses when they interpret daily BP readings, and consider the condition and treatment of patients. Variation *has* to be explained. Not surprisingly, this practice spilled over into the nurses' answers in the questionnaire for the average blood pressure of a particular age group.

How do these insights of 'statistics on the ward' help us understand the nurses' responses in the teaching experiment? Recall that they wanted to explain away variation and found variation and relationship antithetical, while we wanted to introduce them to a fundamental principle of statistical modelling: that data is made up of an explanatory model and random or unexplained variation around this model. On first reading of the nurses' responses, it seemed that their 'intuitions' were falling foul of this important statistical principle, and were based on a deterministic conception of association. Like the university students in the study by Batanero *et al* (1994), it appeared that they were using the many exceptions (or variations) to the blood pressure-age relationship to reject the idea that there was a relationship at all. Or, to restate the problem in the terms used by Rubin and Rosebery (1990), there seemed to be a confusion between the variables in the model and extraneous variables, which meant that presence of the latter made conclusions about any relationships in the model less possible.

Yet our study indicates that the roots of the responses might be rather different: the 'transfer' of the successful strategies of dealing with data over time on the ward to the consideration of population datasets. We use the word 'transfer' with due caution, yet it seems that something like transfer is involved here, but transfer of meanings that embed mathematical ideas to another 'related' situation — which then have to be debugged by reference to that new situation. In suggesting this view of transfer, we are taking the idea out of the strictly cognitive realm, and attempting to reconceptualise transfer in terms of crossing boundaries between practices. Further, we suggest that our current findings throw some light on the cul-de-sac in which the situated cognition perspective seems to leave mathematics. The point is that the nurses *had* constructed an idea of average and variation as part of their routine activity on the ward. What we saw in the teaching experiment was how the mathematical domain could be brought into contact with the nursing domain, and how different tools (in our case, including the technological tools of a particular database program) could catalyse this contact, by exploiting the fact that the nurses could make sense of the new (mathematical) world by using the one tool which they used in their day-to-day practice — the mean blood pressure for an age group. It was this notion that helped them to see the trend in the variability and recapture what they knew of a relationship between age and blood pressure.

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NOTES

[1] *The Tabletop*™ (TERC Inc.) is a database with an appealing graphical interface, which allows students to overcome unfamiliarity with the software quickly.

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