THE SYMBOL-SHOCK – A PROBLEM OF AND IN STATISTICS EDUCATION

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The necessity for stimulating the interest of pupils in mathematics in general and statistics in particular is made clear by the results of surveys. The studies showed, that pupils are tired of mathematics. Mathematics is generally regarded as one of the most unpopular subjects at school. The pupils don't achieve so well in this subject. The causes might originate in several variables. The spectrum of possible causes for these bad results stretches from genetic disposition to deficits in learning behaviour. One of the aspect is that mathematics/statistics use the symbol language. And just exactly this language is what we want to look at more closely. As an essential feature of symbols the fact must be emphasized that they "all mean something other than themselves, that they all point to something besides themselves".

INTRODUCTION

Firstly, a symbol or, respectively, a formula represents *the substitution of an idea*, which it does not possess of itself. The transition of the meaning always takes place from the primary, not perceivable, idea to the secondary, which has a direct effect on the senses and *secondly,* the symbol strives to be shorter and more narrowly condensed than the complex of ideas it represents. This should be demonstrated by the following short example, namely the sum formula.

This process can be iterated, so that, for example:

$$r = \frac{N\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}x_i y_j - \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}x_i \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}y_j}{\sqrt{\left[N\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}x_i^{2} - (\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}x_i)^2\right]\left[N\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}y_j^{2} - (\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}y_j)^2\right]}}$$

Symbols are indispensable components of scientific thought and action. Their meaning and function exist mainly in their ability to reproduce information in a compact form. At the same time they must, by necessity, be decipherable. A central task when teaching is imparting this insight to the pupils and/or students. The problem is, that this notation using symbols could, in certain circumstances, lead to learning problems, as the pupils decide for themselves at the very sight of any symbols, that this is a subject, they will never understand.

It is presumed that pupils don't master mathematics/statistics because they do not understand the learning material, particularly its representation through the use of symbols. Symbols – especially the algebraic symbols I have been doing research on – can trigger meta-cognitive emotions, which I have already termed: the symbol-shock.

WHAT DO I UNDERSTAND BY THE TERM "SYMBOL-SHOCK"?

The phenomenon of the symbol shock and the resulting problems for teaching and learning, especially in mathematics/statistics, are part of the everyday experiences of teachers and pupils of this subject. Faced by the mass of symbolic representations in the notation of mathematics, the pupils often react irritated and, as a next step, they resist and then they block it off. The symbol-shock is a reaction of the pupil towards mathematic symbols, which he is confronted by when trying to solve important mathematic tasks. The symbol-shock is a sign of a psycho-physical state of mental blocking, or respectively, of mentally resisting, which appears at the perception of mathematical symbols.

Seen in the light of the information processing theory, this means the information process concerning the deducible mathematic information is blocked. The person who should be digesting the information is not able to make any use of it, or respectively, cannot digest it in a – for him – satisfactory manner. The mathematic symbols or the mathematic formulas represent for the pupil insurmountable obstacles, and it's clear to the pupil that from the moment a symbol or symbols appear, he will not understand a single thing. This blockade in the information process is accompanied by negative emotions.

In addition to the non-observable, introspective psychic processes under symbol shock, visible patterns of behaviour may accompany this, for instance automatic manipulation, which means the quick and unreflected use of values, and calculating according to the old scheme. The symbol shock usually first appears in mathematic lessons on confronting the pupils with mathematic symbols. And don't forget, the first signs are in the lower classes; the first symbols are the plus sign, minus sign and equal sign. Those pupils who have suffered from it during their schooling learn how to handle the experience. The already mentioned behaviour pattern, *manipulation according to the same old way* is one of these ways of handling the situation. A learner who, at the moment he recognizes the symbols, takes the decision that the task is not to be solved, need not necessarily be automatically inactive in mathematic operations. The learner will try – as far as he has already made some experiences with calculation operations – to take control of the learning material mechanically. It is definitely possible for a learner to be in a position to use mathematical symbols or formulas, by mechanically using rules for the manipulation of these symbols. The learner doesn't even try to understand the mathematical content of the present problem, but produces results that comply with the superficial formal requirements. He manipulates mathematic symbols without understanding them, possibly as a result of a symbol shock.

According to my research results, a symbol shock can arise in the following situations:

- in situations where for the active person an important situation is at hand.
- in situations when for the active person the task is not completely new, but he is not completely acquainted with it: he doesn't know enough to master the task, but he knows enough to feel unsure.
- in situations where the active person has the impression something is difficult, or, respectively, is neither to be perceived nor understood.

The occurrence of the symbol shock presupposes that something has been perceived, or is perceived. Generally speaking, for the perception of symbolic objects, a lot of abilities have to be learnt. I would like to mention Piaget in this context. For the correct reading and interpretation of mathematic symbols, a high level of ability is demanded, through which the further processing of the original perception data is largely controlled by symbolic cognitive processes. In the following, the various levels should be shown at which a mathematic formula can be perceived. The levels of perception that are necessary to understand a formula should be represented here in short with the correlation coefficient "r".

PERCEPTION AREAS WITH MATHEMATICAL SYMBOLS

With the letter "r" we have an "unmistakable symbol", where the letter "r" as a representative element has a quite clearly defined mathematic meaning. The drawing of a diagram of the total mathematic content of "r" is both of a pictorial as well as a non-pictorial nature. "r" is a mathematic correlation measure between two characters, represented by the formula just mentioned. For "r" it applies that: *Elements out of the mass of perceivable forms are assigned to a single element out of the mass of non-perceivable contents*. The perception of "r", it is assumed, can take place at different levels, of perception as shown in Figure 1.

Perception results (the perceived meanings of "r") are always felt to be insufficient and negative when the person knows that in the expression "r" there is more – for them valuable - information contained, which however remains untapped for them. Such "knowledge gaps", perceived by the learner as unfillable, can become one of the triggers of meta-cognitive feelings, and thus of the symbol shock.

P1 "r" is recognized as a letter of the Roman alphabet
P2 "r" is recognized as a mathematic symbol, that is as the representative for a mathematic fact that is not known closer.
P3 "r" is recognized as a symbol for a "mass of relations". A "mass of relations" in this case is only a word-wrapping without any filling as regards the content.
P4 "r" is recognized as a representative of the term
P5 "r" is understood as an instruction for mathematic acts, which are depicted in short by the mathematic notation symbol. For instance-

$$r = \frac{s_{xy}}{s_x s_y} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2}}$$

Σxi means- "add the single xi". The person concerned can carry out the mathematic instruction.
P6 "r" is seen as an illustration of empirical facts, that means it is understood that the calculations contained in the formula adequately reflect the empirical state of affairs waiting to be worked on.
P7 "r" is understood with the mathematic idea which it stands for.

Figure 1. Perceptions of "r".

FOR THE TEACHER IT IS IMPORTANT TO KNOW WHAT LEVEL OF PERCEPTION THE LEARNER IS AT.

Conclusions about the learner's basic level of perception can be drawn, for example by analysis of their mistakes. A purely mechanic reaction is to be expected at perception level p5. Here the learners are able to recognize and carry out the fitting mathematical calculations, but all perception of the further levels, and thus the knowledge of what the symbol actually stands for, remains obscure. One can say, that the expert operating at the perception level p7 perceives what is depicted by the symbol and, as a result, grasps the deepest meaning of the mathematical symbol. The questions which now arise are: *How can a symbol shock be avoided, and how can pupils already suffering from symbol shock be helped?*

Symbol shock is brought about, above all, when formulas are just presented as finished products. The forms into which the abstraction process leads (naturally enough with good reason), appear to the pupils like something from another planet. If a sense of form is not inculcated from the beginning - then symbol experience is already shocking on the surface. The confrontation with mathematical statistics symbols can be guided by meta-cognitive feelings – including experiences of symbol shock – which stand in the way of a successful mastering of the task.

It is a necessity to have ways of instructing the pupils that then lead to an *understanding of the symbols used*, and not waiting till the pupils have suffered a symbol shock, and then having to find a remedy for damage already done.

As an illustration I would like to give you a report about a project I carried out one year ago. The starting point was, that the symbol shock prevents many learners from successfully coming to terms with mathematic tasks. The aim of the project was to provide the pupils with an unprejudiced approach to mathematic symbols and to tackle mathematic tasks with deeper insight and greater self-confidence. Last but not least, the project was to help dismantle essential obstacles which hinder the pupils' engagement in the field of mathematics.

How did I approach and go about this? The test group consisted of eight ten-year-old girls who had only had basic arithmetic at school. Contrary to previous projects, however, they were all well used to using PCs, and were therefore able to carry out the tasks at the PC without any problems. As in the years before, the project was called "Secret Languages, Secret Signs".

What do the pupils have to do? Firstly, the pupils are asked to write their names and find symbols for them. Secondly, the pupils have to demonstrate a selection of symbols they are already acquainted with: traffic signs, etc. Thirdly, the pupils learn secret languages such as Morse code. They soon discover, for instance, that whether you use an upper-case dash (-) or a lower-case one (_) is irrelevant, it is only important to keep to the same pattern. Fourthly, they soon develop their own secret codes. Fifthly, there is then a competition to decode secret languages and they learn how to crack the other pupils' codes. Finally, there is a race against the clock to try and solve codes. The time is noted on the blackboard. Amazingly, in this case all the children understood the term "average", and they soon hit on ways of finding the average.

The children were then asked to find a basic prescription for finding averages. The presentation shown in Figure 2 was made.

Take the first girl's result and add it to the second girl's result and add the third girl's result and add…the result of the last girl and then divide it by the number of girls.

Figure 2. Basic Prescription For Finding Averages.

They quickly discovered that the whole thing could be abbreviated in the presentation. For example, they suggested taking the letter A for average... . For the first girl you could take the abbreviation: A = 1. girl + 2. girl + last girl/divided by the number of girls.

Then they hit on: A = 1. G + 2. G. …+ L. G./divided by the number of girls (First girl+second girl…+last girl)

They became a bit unsure of themselves when I asked them to show me how we could calculate the results with boys. But even here they found a speedy solution. *Okay, then we'll use a B instead of a G* and asked: *what happens if it's a group of adults? Why not S for seniors?* The provocation ended by their saying: *Wouldn't it be enough to say R for result?* Then you could always use 1.R + 2. R + ...l. R, and then divide by the number of results.

We then tested our script to see if it always worked, e.g.: for 3 persons: 1.R+2.R+3.R / 3 etc. They noticed a connection between the number of the last result and the number by which the sum was to be divided as in Figure 3.

1.R+2.R+3.R / 3
1.R+2.R+3.R+4.R+5.R/5

Figure 3. Comparison of Results.

In-between they were informed, that in mathematics the number for the index is written lower case. R_1+R_2+R_{last} / to be divided by the same number as the last one.

The script: $\frac{R_1 + R_2 + R_{last}}{last}$ didn't bother them. At the start of the project they had already heard that there are various ways of writing, and so why not use the fraction sign if it means the same as the division sign. It should be noted, that this way of teaching demands taking a lot of time before the effects are seen. On the other hand, a transfer does take place. Whether somebody has understood statistics symbols or not can be seen in his putting the formula into practice, and on the other hand his being able to convert practice into formula.