YOUNG CHILDREN'S PROBABILISTIC THINKING

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Task-based interviews were used to investigate some aspects of probabilistic thinking in 74 children (aged 5 to 12 years) from three different Australian schools, who had received little or no instruction in this topic. As expected, there was evidence of the development of probabilistic reasoning with age, but analysis of the children's responses and explanations revealed a variety of decision making strategies. Further comparisons between the mathematical structure of each task and the mathematics inherent in the strategies used by the children, suggest that the design of the task, including the actual materials chosen, may be a crucial factor in teaching basic probability concepts to children.

A recent direction in the research on the development of probability concepts in children is the attempt to develop a comprehensive framework that can be used to describe and assess children's understanding, which in turn can provide guidance for teaching (Watson and Collis, 1994; Fischbein and Schnarch, 1997; Jones, Langrall, Thornton, and Mogill, 1997). Hopefully this will also make it easier to link together the information gathered from the numerous specific studies, and facilitate the mapping of the relationship between basic probability concepts and other mathematical concepts.

The study described in this paper is one that has attempted to take advantage of the fairly unique opportunity to examine intuitive understandings of probability concepts in school age children. This situation has arisen due to the absence of the topic of Probability from the New South Wales (Australian State) primary mathematics syllabus. The children for the study were drawn from three different schools and were nominated by teachers as being fairly representative of the school's mathematical performance. A set of games in which the children were asked to make certain choices and explain their reasoning were designed and used in one-to-one tape recorded interviews. The set of games evolved during the study so that none of the three schools completed exactly the same interview protocol. The sample size for each of the five research tasks ranged from 25 to 74 children. One task, *Bears in a Box,* was complete by every child in the study.

RESEARCH QUESTIONS

The key research questions for this exploratory study were:

1. What strategies do children utilise for making judgements in different types of probability tasks?
2. What is the relationship between these strategies and the type of probability task?
3. Can the children's responses be classified into the expected developmental stages of non-probabilistic thinking, estimation of probability and quantification of probability?

Crucial to the design of the interview protocols was the interpretation of '*different types of probability tasks'*. Probability tasks in the study were designed by varying the following four factors in such a way as to use most combinations of factors, and assess a range of probability concepts.

The Type Of Random Generator

Two types of random generator were chosen for the study, each modelling its sample space in a physically different way. The Numerical Model consisted of a set of discrete objects (small plastic bears) placed in a container (a box or a jar) for drawing out without looking into the container. The Spatial Model consisted of spinners with coloured areas marked as sectors of a circle, and a 'spinnable' pointer attached to the centre were used. The choice and design of the random generators was influenced by the work several people. J. Truran's (1994) classification system of random generators used in research provided a structure for quite detailed analysis of the research tasks. Hoemann and Ross's (1971) exploration of the validity of using spinners in certain ways to assess probability concepts prompted the inclusion of the *Transfer Task* in the study. K. Truran's (1995) findings in regards to young children's perceptions of random generators assisted with the planning of questions and interpretation of children's responses. The work of Fischbein and his colleagues (for example Fischbein, Pampu and Manzat, 1970), highlighted the need to include a task that would prompt numerical thinking, particularly involving ratios.

Mathematical Structure of the Sample Space

This is a major factor in the design of a task and involves several aspects; the total size of the sample space, the number of components (such as colours), and the ratio of the components (such as the number of each colour).

Nature of Comparison

The questions used during each task were largely determined on the basis of the type of comparison the child was required to make. This, together with the structure of the particular sample space, determined the probability concepts being assessed. For example, some questions were designed to assess the children's understanding of equal likelihood, while others were directed at the notion of 'most likely'. The three main types of comparison included in the tasks were: a) Intra-Sample Space involving the comparison of the components within a single sample space, such as determining what colour bear would be most likely to be drawn from the box in *Bears in a Box*.; b) Inter-Sample Space involving the comparison of two or more separate sample spaces, such as determining the best out of four different spinners for spinning Red in the *Racing Car Game*; and c) Comparing Two Different Types of Random Generators, relating the numerical model to the spatial model random generators, which was the essence of the *Transfer Task*.

Type of Response

In all tasks, two types of responses were elicited, a simple choice and a stated reason, possibly including a physical demonstration using the equipment.

BRIEF DESCRIPTIONS OF THE TASKS

Task 1: Bears in a Box - Four small coloured bears (3 of one colour, 1 of another colour) are placed in a box. The child is asked to say which colour is most likely to be drawn out, and why the choice has been made. The child draws out a bear and then replaces it. This is repeated 5 times, with a display kept of each outcome.

Task 2: Non-replacement - This is similar to the first task except the bear is not replaced after each draw.

Task 3: Racing Cars - Four spinners of differing construction are used to play a game where coloured discs are moved along a race track. The child is asked to make various choices in regards to the most likely winner using a particular spinner, or the best spinner to use to get a certain result. Reasons are sought for each choice.

Task 4: Transfer - The child is asked to place coloured bears in the box to replicate the colour ratios of the spinners from the Racing Car game and to explain how they decided on the number of bears of each colour.

Task 5: Proportions - Two jars of various mixtures of red and yellow bears are displayed to the child, who is asked to choose the jar that will give the better chance of drawing out a red. The coloured bears are lined up outside the jars to facilitate the child's choice and explanation.

RESULTS

The most basic level of examining the results was to simply categorise the responses as 'correct' or 'incorrect'. Incorrect responses indicated non-probabilistic thinking and correct responses indicated either appropriate estimation of probability or quantification of probability.

Table 1. Percentage correct in each age group across all tasks

Age Group	Choices	Reasons
5/6years	64.9	45.9
7/8 years	81.2	69.5
9/10 years	92.3	85.7
11/12 years	92.4	85.8

In general, the results across all tasks showed a clear increase in undersatnding of basic probability concepts with age, though it can be seen in Table 1 that there was, overall, very little difference between the performance of 9 and 10 year olds and the 11 and 12 years olds. Another overall trend was that, for each age group, more children were able to make correct choices in the tasks than give correct reasons for their choices.

THE CHILDREN'S STRATEGIES

In the easier tasks, Bears in a Box, Non-replacement and Racing Cars, the majority of children over 6 years successfully used estimation strategies to make and justify their probability judgements. Little quantification was voluntarily applied and indeed quantification was not really required to produce a correct response in these tasks. A statement such as, "There are more blue than red" communicates the reason for selecting blue as being more likely to be drawn from a box, just as effectively as does the statement, "3 out 4 are blue, but only 1 out 4 is red". Table 2 provides a summary of the percentage of children who applied each of three levels of responses (or strategies) for each of the first three tasks, in each of the age groupings. It can be seen that most children used estimation of probability in each of the tasks. Task 3, the *Racing Car* game using spinners, encouraged greater use of fractions to describe the size of the spinner sector, and hence the likelihood of a certain outcome. While the children were usually not consciously calculating a probability, they were deliberately using numbers to indicate the chance of an event occurring.

Table 2. Percentage of each age group using each strategy for each of Tasks 1, 2 and 3.

	Non-probabilistic			Estimation			Quantification		
Age Group	Task 1	Task 2	Task 3	Task 1	Task 2	Task 3	Task 1	Task 2	Task 3
5/6yrs	73.0	65.0	25.0	25.4	30.0	60.4	1.6	5.0	14.6
7/8yrs	28.1	46.2	1.9	59.6	50.0	75.0	12.3	3.8	23.1
9/10yrs	15.8	25.0	3.8	66.7	70.8	67.3	10.5	4.2	28.8
11/12yrs	22.2	9.1	0.0	71.1	77.3	59.1	6.7	13.6	40.9
Total Proportion	36.5	35.9	7.7	54.1	57.6	65.8	7.7	6.5	26.5

However, the *Transfer Task* and the *Proportions Tasks* were more challenging, elicited a more complex range of responses, and included more attempts at quantification than the easier tasks. These two tasks demanded higher order thinking, involved more sophisticated mathematics and prompted the invention of quantification strategies especially for the task. Although the strategies used by the children could be grouped under the headings of *Non-Probablistic, Estimation and Quantification*, these categories were too broad to adequately describe the full range of strategies invented by the children. Tables 3 and 4 list the main strategies used by the children in each age group and are listed in increasing order of mathematical sophistication. (The strategies used by the children in the *Transfer Task* are described in detail in Way, 1996). In the *Proportions Task* there was a clear tendency for children to revert to the least demanding strategy that might be successful in making the correct choice (Way, 1997).

Table 3. Task 4 Transfer Strategies: Percentage using each Strategy

Age Group	Non-comparison Strategy	Measurement Strategy	Ordering Strategy	Fractional Strategy
5/6	50.0	41.7	8.3	0.0
7/8	0.0	61.5	38.5	0.0
9/10	0.0	23.1	30.8	46.2

11/12	10.0	0.0	30.0	60.0
Total Proportion	14.7	35.4	27.1	25.0

Table 4. Task 5 Proportions 1 and 2: Games 1 to 7: Total Percentage using each strategy

Strategy Category	5/6 yrs	7/8 yrs	9/10 yrs	11/12 yr
1. Idiosyncratic	25.3	24.2	4.5	4.1
2. Comparing favourable	51.8	32.3	24.2	20.8
3. Comparing unfavourable	7.2	16.1	10.6	8.3
4. Subtractive	16.9	25.8	40.9	41.6
5. Proportional	0	1.6	18.2	29.1

IMPLICATIONS FOR TEACHING

Something that has become obvious during this study, is the significance of the impact that the particular random generator has on the way a child responds to a probability activity. A second crucial ingredient is the nature of the comparison that is expected in a task. While the *Bears in Box*, *Non-replacement*, and *Racing Car* tasks provided sufficient challenge to the 5/6 year olds and many of the 7/8 year olds, these activities only elicited low level responses from the older children. Most children demonstrated strong intuitive insights and self-taught skills. The *Transfer* and *Proportions* tasks encouraged the children to draw on a range of other mathematical knowledge and skills and to use these to invent strategies to solve the problem rather than rely on intuition. This highlights the need, not only to develop supporting understandings in fractions, ratio and proportion, but the power of certain probability tasks to extract this type of mathematical thinking from children. This, I feel is the real potential for mathematical learning that probability topics hold for primary age children. Considering the extreme lack of emphasis on probability, data, fractions and proportions in the NSW primary mathematics syllabus, it is somewhat surprising that the children managed the level of quantification that they exhibited in the tasks.

Unfortunately, the data from this study cannot be used to trace the use of strategies by individual children across all of the tasks, because no child completed the entire set of tasks. A rewarding future study, perhaps in the form of several case studies, would be to conduct an instructional experiment, where the child's initial strategies could be mapped, then the development of the strategies monitored during a period of 'instruction'. The child's skills in other related areas of mathematics would provide useful background data that could be compared to the mathematics utilised by the child in the probabilty task strategies.

REFERENCES

Fischbein, E., Pampu, I. and Manzat, I. (1970). Comparison of ratios and the chance concept in children. *Child Development, 41*, 377-389.

Fischbein, E, and Schnarch, D. (1997). The evolution with age of probabilistic intuitively based misconceptions. *Journal for Research in Mathematics Education, 28(1),* 96-105.

Hoemann, H., and Ross, B. (1971). Children's understanding of probability concepts. *Child Development*, 42, 221-236.

Jones, G. A., Langrall, C. W., Thornton, C. A. and Mogill, A. T. (1997). A framework for assessing and nuturing young children's thinking in probability. *Educational Studies in Mathematics, 32*, 101-125.

Truran, J. (1994). Diagnosing children's understanding of probability. . In G. Bell, B. Wright, N. Leeson and J. Geake (Eds.) *Challenges in Mathematics Education: constraints on Construction: Proceedings of the Seventeenth Annual Conference of the Mathematics Education Research Group of Australasia*.. (pp.623-630). Lismore, Australia.

Truran, K. (1995). Animism: A view of probability behaviour. In Atweh, B. and Flavel, S. (Eds.) *Galtha: Proceedings of the Eighteenth Annual Conference of the Mathematics Education Research Group of Australasia*.. (pp. 537-542). Darwin, Australia.

Watson, J., and Collis, K. (1994). Multimodal functioning in understanding chance and data concepts. In J. da Ponte and J. Matos (Eds). *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education.* (pp.369-376) Lisbon, Portugal.

Way, J. (1996). Children's strategies for comparing two types of random generators. In L, Puig and A, Guitierrez (Eds.), *Proceedings of the Twentieth International Conference for the Psychology of Mathematics Education.* Vol. 4.(pp. 419-426) Valencia, Spain.

Way, J. (1997). Which jar gives the better chance?: A study of children's decision making. In Biddulph, F. and Carr, K. (Eds). People in Mathematics Education: *Proceedings of the Twentieth Annual Conference of the Mathematics Education Research Group of Australasia*. (pp. 568-575). Rotorua, New Zealand.