STUDENTS' (GRADES 6-8) UNDERSTANDING OF GRAPHS

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This study focused on middle grades students' reasoning in response to questions about a bar graph. There were three levels of questions, and responses were coded to identify knowledge of structural components of graphs, accuracy of graphical-perception tasks, and development of graph schema. At each grade level, students (N=76 in grade 6; N=71 in grade 8) were taught by a single teacher who had been certified as a statistics educator. The teachers taught an experimental graphing unit, with a paper-and pencil task administered before and after the unit. Students seemed to learn to coordinate the components of graphs, performed graphical-perception tasks more accurately, and developed more coherent graph schema. There was some evidence to support the existence of a (perhaps developmental) sequence of reasoning strategies.

This study investigated the ways that students in the middle grades (grades 6-8 in USA) interpret data presented in a bar graph. Of particular interest was the nature of the reasoning that students used as they explained their answers to questions.

THEORETICAL PERSPECTIVES

Two theoretical perspectives framed this study. First, a number of authors (Bertin, 1973; Curcio, 1987; McKnight, 1990; Wainer, 1992) have characterized the kinds of questions about data in graphs and, to some extent, the nature of the responses to these questions. Three levels of questioning emerge: an elementary level focusing on extracting data from the graph, an intermediate level involving interpolating and finding relationships in the data shown in the graph, and an overall level involving extrapolating from the data and interpretation of the relationships identified in the graph.

Second, structures are emerging for the study of graphical understanding. Although there is not yet universal agreement, there is overlap in the features that command the attention of theorists. Kosslyn (1989, 1994) has proposed language for discussing four structural components of graphs. The *framework* (e.g., axes, scales, grids, reference markings) provides information about what kinds of measurements are being used and what things are being measured. The simplest frame-work has an L shape; one leg, the x-axis, stands for the things being measured and the other, the y-axis, for the measurements. Visual dimensions called *specifiers* are used to represent data values. Specifiers are the lines, bars, point symbols, or other marks that specify particular relations among the things represented within the framework. In an L-shaped framework, each leg of the framework bears a *label* naming the type of measurement being made (a dependent variable) or the entity to which the measurement applies (an independent variable). Labels include coding techniques for representing category membership. The title of the graph is itself a kind of label. The *background* of a graph includes any coloring, grid, or picture over which the graph may be superimposed.

Cleveland and McGill (1984) have identified ten graphical-perception tasks that form the basic perceptual judgments that a person performs to decode visually quantitative information encoded on graphs (e.g., specifiers), though it is possible that these tasks might be grouped into fewer distinct categories. The tasks are ordered, based on what is known about the accuracy with which a person performs them, from most accurate to least accurate; some tasks are identified as being at the same levels. For example, finding positions on a common aligned scale (e.g., bar graphs) can be processed more easily and more accurately than determining area (e.g., pie charts).

Pinker's (1990) theory of graph comprehension is based upon general perceptual and cognitive assumptions. His notion of *graph schema* is important here. A graph schema includes a person's knowledge of relationships between aspects of the visual description as they are connected to aspects of the conceptual information. A graph schema is an active, interrelated knowledge structure, a network of nodes and relations, embodying knowledge of what graphs are for and how they are interpreted. It is useful for translating the information found in the visual description into conceptual information and directing the search for desired pieces of information. A graph schema contains three key pieces of information: the display's pictorial content of some objects or geometrical figures, such as bars or lines, which are described in terms of visual attributes; the L-shaped framework of a coordinate system, specifying the ratio of magnitudes of attributes; and the textual material which specifies the real-world referents.

METHOD

Subjects

Subjects were students in grade 6 (N = 76) from three classes taught by one teacher and students in grade 8 (N = 71) from three classes taught by a different teacher in a different middle school in North Carolina. The students had had little experience with statistics prior to this study. Both teachers were "statistics educators" from the Teach-Stat project (Friel and Bright, in press). These teachers had participated in a three-week statistics education inservice, used the techniques from this inservice in their own teaching, and co-taught two or more statistics education inservice programs to other teachers. Thus, their knowledge of statistics content and pedagogy was considerably greater than that of most middle grades mathematics teachers.

Instrumentation

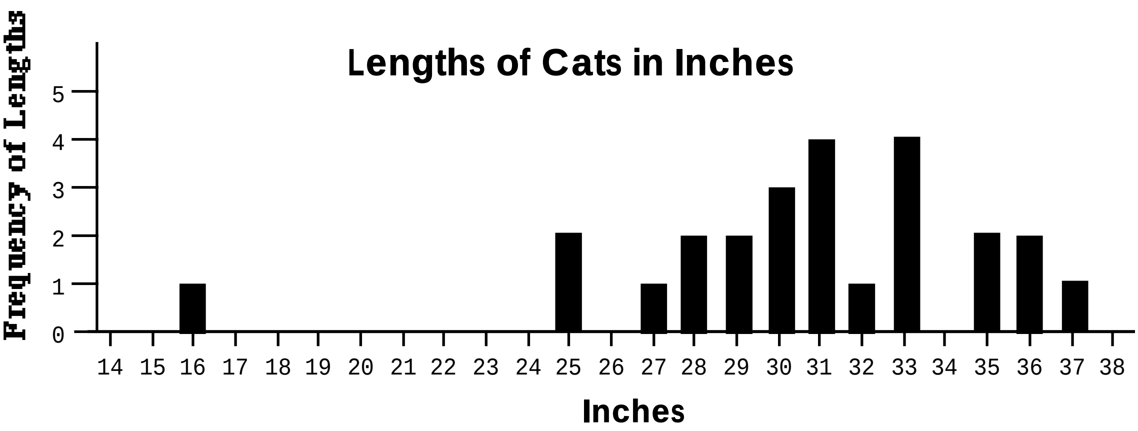
A paper-and-pencil task (Figure 1) was used; the four questions reflected the three levels of questions described earlier, with Questions 2 and 3 used to assess students at the second level. Answering these questions calls on knowledge of structural components of graphs, graphical-perception tasks, and graph schema.

Instruction and data gathering

Both classroom teachers organized instruction around the same unit on graphing (Friel and Bright, 1995), developed specifically to highlight connections between pairs of graphs. The unit was not provided as a definitive model of instruction. Rather, its use was intended to give students opportunities to gain experience with the process of statistical investigation and with some key concepts (including graphs) in statistics. The task was administered both before and after instruction during October and November, 1994.

Figure 1. Task: Lengths of Cats

A group of students has been investigating information about their pets Several students have cats. They decided to collect some information about each of the cats. One set of data they collected was the length of the cats measuring from the tips of the cats' noses to the tips of their tails. Here is a bar graph showing the information they found:



1. How many cats are 30 inches long from nose to tail? How can you tell?
2. How many cats are there in all? How can you tell?
3. If you added up the lengths of the three shortest cats, what would the total of those lengths be? How can you tell?
4. What is the typical length of a cat from nose to tail? Explain your answer.

RESULTS

Based on analysis of the task within the theoretical perspectives discussed earlier, coding categories (Figure 2) were created for the questions; category numbers are arbitrary. All responses were double coded to assure reliability of coding. (Because of space limitations, data are reported only for questions 2 and 3.)

When categories generated unique answers, they were split into pairs (Figure 3); the "a" part reflects a response accompanied by an appropriate explanation, and the "b" part reflects a response either without any explanation or with an inappropriate explanation. In each of the tables in Figure 3, the totals down the right side are pretest totals, and the totals across the bottom are posttest totals. Since the emphasis of this analysis was on students' reasoning, some leeway was allowed in categorizing numerical responses. It seemed more important to categorize evidence of reasoning than skill at arithmetic. Of course, no leeway was allowed for the "b" part of a pair of categories.

For Question 2, 27 out of 76 students (about 36%) in grade 6 and 40 out of 71 students (about 56%) in grade 8 gave pretest and posttest responses that were coded in the same category. For Question 3, the corresponding data were 22 out of 76 students (about 29%) in grade 6 and 18 out of 71 students (about 25%) in grade 8. The most common change in categorization seems to have been shifts *from* incorrect to correct reasoning (columns 1a and 1b, below rows 1a and 1b).

Figure 2. Coding Categories and Sample Responses for Questions 2 and 3

Question 2: How many cats are there in all? How can you tell?

1: Properties of the graph; considers both the range of data and the frequencies
  25 cats in all. You add all numbers represented by bars together.
2: Focus on range of data or indicated range on x-axis
  37 because it is the last number marked.
3: Uses frequencies of heights of bars
  12 cats, I could tell because there are twelve bars on the graph.
4: Incorrect or incomplete or not understandable
  24, because you add them all up and see that it is 24.

Question 3: If you added up the lengths of the three shortest cats, what would the total of those lengths be? How can you tell?

1: Choose correct lengths of 16, 25, and 25 to get 66
  16 one and two 25. Read the first two bars.
2: Added 16 and 25 to get 41, ignoring that there are two cats of length 25
  It would be 41. You add up the 2 shortest cats which are 16 in + 25 in = 41 in.
3: Added lengths associated with three left-most bars of height one (16+27+32=75)
  The length of the 3 shortest cats is 73 inches in legth (sic). I added 16 to 27 and 32 to 41 [which is the incorrect sum of 16 and 27] and got my answer 73 inches.
4: Added lengths associated with three right-most bars of height one (27+32+37=96)
  27, 32, 37, the graph showes (sic).
5: Added lengths associated with all four bars of height one (16+27+32+37=112)
  112, I can tell by adding up the inches that are the shortest shown on the bar graph.
6: Identified or added frequencies of shortest bars to get answer of 1, 3, or 4.
  3, all you got to do is count the number of sort (sic) black lines.
7: Added lengths associated with three left-most bars (16+25+27=68)
  68 inches in all. I looked at the first 3 bars and added the inches.
8: Added frequencies for three left-most bars (1+2+1=4)
  4 go to the left side of the scale find the first three marks & add them up.
9: Incomplete or not categorizable
  3, I added from the side of the chart.

DISCUSSION

For both grades, across the two administrations of the task, there was an increase

(Figure 3) in the number of students responding correctly using the properties of a graph and a decrease in the number of students interpreting the heights of the bars as lengths of cats. These results are consistent with the notion that students learned to coordinate the components of graphs (e.g., framework, specifiers, labels), performed graphical-perception tasks more accurately, and developed more coherent graph schema. Students seemed better able to understand the nature of data reduction evident in the graph in the sense that they moved away from interpreting each bar as representing a single cat. Too, the number of non-categorizable responses seemed to decrease, suggesting that students internalized a vocabulary for discussing their reasoning. These skills are critical ones for appropriate interpretation of data in graphs.

In general, the questions seemed somewhat easier for grade 8 students than for grade 6 students, especially on the first administration. This may reflect the greater experience of grade 8 students with graphs. Similarly, Question 2 seemed easier than Question 3, even though they were at the same level (e.g., Bertin, 1983, Curcio, 1989). The difficulty with Question 3 seems to have been in knowing how to identify the shortest cats from the graphical presentation of the data.

The largest subset of students who gave identically coded answers on both administrations were students who gave correct answers both times. It seems important that among students who gave incorrect answers on the first administration, the largest shift was to correct reasoning. Instruction did appear to be effective at helping them restructure their knowledge of graphs.

Although the large number of categories for Question 3 makes pattern finding difficulty, there was a suggestion about how reasoning might develop. For grade 6, 16 pretest and 15 posttest responses were categorized as 7a/7b. For grade 8, 16 pretest and 13 posttest responses were categorized as 7a/7b. However, few students gave both pretest and posttest responses that were categorized this way. Focusing on the three left-most

Figure 3. Summary of Responses for Questions 2 and 3

Grade 6, Question 2

Code	*1a*	*1b*	*2*	*3a*	*3b*	*4*	*Totals*
1a	13	5	0	0	0	1	19
1b	6	10	0	0	1	1	18
2	0	1	0	0	0	0	1
3a	8	5	0	1	2	3	19
3b	1	1	0	0	0	1	3
4	6	7	0	0	0	3	16
Totals	34	29	0	1	3	9	76

Grade 8, Question 2

Code	*1a*	*1b*	*2*	*3a*	*3b*	*4*	*Totals*
1a	16	5	0	0	0	1	22
1b	3	18	0	0	1	4	26
3	0	1	0	0	0	0	1
3a	0	2	0	2	1	3	8
3b	0	1	0	1	0	0	2
4	2	6	0	0	0	4	12
Totals	21	33	0	3	2	12	71

NOTE: Codes down the left side are pretest codes; codes across the top are posttest codes.

Grade 6, Question 3

Code	*1a*	*1b*	*2a*	*2b*	*3a*	*3b*	*4a*	*4b*	*5a*	*5b*	*6a*	*7a*	*7b*	*8a*	*9*	*Totals*

1a	8	2													5	15
1b	2	1		1									1		1	6
2a																
2b	1															1
3a											1	2			1	4
3b	1															1
4a						1						1				2
4b										1	1				1	3
5a															1	1
5b										1						1
6a		2	1								2	2				7
7a	5	1				1					1	3	1		3	15
7b													1			1
8a	1															
9	2				1	1		1			3	1	3		6	18
Totals	20	6	1	1	1	3		1		2	8	9	6		18	76

NOTE: Since codes 6a and 8a generate similar answers, categories 6b and 8b could not be coded.
NOTE: Codes down the left side are pretest codes; codes across the top are posttest codes.

Grade 8, Question 3

Code	*1a*	*1b*	*2a*	*2b*	*3a*	*3b*	*4a*	*4b*	*5a*	*5b*	*6a*	*7a*	*7b*	*8a*	*9*	*Totals*
1a	12	1									1				1	15
1b		1											1		3	5
2a																
2b																
3a					1							1				2
3b		2										1	1			4
4a															1	1
4b															1	1
5a											1				1	2
5b															1	1
6a	1										1	1			1	4
7a	5				2	2					1		1		3	14
7b	1												1			2
8a																
9	3	2				4	1				2	3	3		2	20
Totals	22	6			3	6	1				6	6	7		14	71

NOTE: Since codes 6a and 8a generate similar answers, categories 6b and 8b could not be coded.
NOTE: Codes down the left side are pretest codes; codes across the top are posttest codes.

bars seems to hold considerable appeal for students, independent of instruction. In both grades, however, the most common shift *away from* category 7 was *to* category 1, and the most common shift *to* category 7 was *away from* the "uncategorizable" category 9. This suggests that category 7 reasoning may act as a transition from uninterpretable responses to correct reasoning.

The data in this study support the notion that emerging theoretical structures for interpreting students' responses provide a way for both researchers and teachers to begin to understand students' reasoning about graphs. Further study of the patterns of reasoning is needed, along with more trials of instructional materials which are based on these emerging theoretical structures

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