

## WHAT IS STATISTICAL THINKING?

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*This paper begins an exploration of “statistical thinking”. We take as our starting point the conception that the ultimate goal of (applied) statistical investigation is learning in the context sphere of the problem and that “statistical thinking” is the interplay between the statistical and the contextual. This macro-level discussion of statistical thinking draws upon interviews with statistics students and professional statisticians. We concentrate on broad aspects of thinking skills required in applied statistics, indicate peculiarly statistical aspects, and make recommendations for further work. Along the way we discuss contributions of the quality movement to statistical thinking.*

### INTRODUCTION

The desire to imbue students with statistical thinking has led to the recent upsurge of interest in incorporating real investigations into statistics education. However, rather than being a precisely understood idea or set of ideas, the term “statistical thinking” has acted more like a mantra that evokes things that are understood at a vague, intuitive level, yet are largely unexamined. Statistical thinking is the statistical incarnation of “commonsense”. “We know it when we see it”, or perhaps more truthfully, its *absence* is often glaringly obvious. It is often used in contrast to the mechanical application of statistical techniques. In the quality area, the term has been used with a much more specific purpose — as a code under which a particular set of insights about how processes operate has been promoted. In the words of Snee (1990) (see also Britz et al. 1997), statistical thinking is defined as “*thought processes*, which recognise that variation is all around us and present in everything we do, all work is a series of interconnected processes, and identifying, characterising, quantifying, controlling, and reducing variation provide opportunities for improvement.” Outside the quality area, “thinking statistically” has somewhat wider ramifications than this.

The usual panacea for teaching students to think statistically is, with apologies to Marie-Antoinette, “let them do projects”. Although this enables students to experience more of the breadth of statistical activity, it is not enough. The cornerstone of teaching in any area is the development of a theoretical structure with which to make sense of our experience, to learn from it and transfer our insights to others. An extensive framework of statistical models has been developed to deal with technical aspects of the design and

analysis that are applicable once the problem and variables have been defined and the basic study design has been decided. An enormous amount of statistical thinking must be done, however, before we ever reach this stage. Even the interplay between data and context at the analysis stage — in which we continually shuttle between the statistical data sphere, where we go to find something out, and the context sphere where we return to ascertain the meaning of what we have seen — is simply not catered for. This is something we should begin to remedy.

The authors have begun trying to identify important elements from the rich complexity of statistical thinking. In addition to the literature and our own experience, our discussion draws upon intensive interviews with students of statistics and practising professional statisticians. This paper also highlights the contributions of the quality movement to this area. We have come to believe that the combination of process models (c.f. quality improvement theory) with worry and trigger questions (systemise what you can, stimulate what you cannot) provides an excellent analytical environment for building models for statistical thinking.

#### CONCEPTION OF APPLIED STATISTICS

*The ultimate goal of statistical investigation is learning in the context sphere.*

The primacy of the problem-context domain must be recognised. Statistics is a servant discipline. Every problem an applied statistician works on is embedded in a larger problem, the “real problem”. Surrounding this larger problem is a body of “context” knowledge. Statistical investigation is carried out because people deem their context knowledge insufficient for their desired uses, be it as a basis for decision making and action or simply for understanding. Statistical investigation is used to expand the context knowledge-base. Its goal is learning in order to produce an improved context-matter understanding. Learning (in general) is much more than collecting information, it involves synthesising the new ideas and information with existing ideas and information. The “statistical thinking” that interests us consists of the generic thinking processes which *should* take place where statistical methodology meets a real-world problem. We now map out a tentative overview of statistical thinking under 5 main headings: foundations; modes of thinking; attitudes or dispositions; constraints; and techniques.

#### FOUNDATIONS OF STATISTICAL THINKING

We consider the following four elements as laying the foundations for “statistical thinking”: taking account of variation; constructing and reasoning from models; having some knowledge base in both the statistical domain and the context domain; and synthesis or integration.

First, recognising of the role of variation (whether inherent or introduced by the observational process) and acting in a way that takes that variation into account, is what makes an activity “statistical”. Although we do not have the space here, serious consideration of statistical thinking requires constructing a holistic high-level view of “statistical approaches” to variation. Second, our thinking is model based; we construct, adapt and enhance models and make deductions or predictions from them. Our formalised statistical models are a tiny subset of the models we use, as are the models of context-matter theory. We have a myriad of internal mental models of our own creation (largely subconscious) for how systems work and fit together. We extract and form “meaning” in terms of those models. Third, we need some statistical knowledge and some context knowledge to engage in statistical thinking. Finally, the raw materials on which statistical thinking works are statistical knowledge, context knowledge and the information in data. The thinking itself is the synthesis of these elements to produce implications, insights and conjectures.

#### MODES OF THINKING

At any one time, the thinker will probably be involved in one of the following modes of thought. (There is some overlap between these modes.)

- Strategic (anticipating and planning)
- Interrogating (actively acquiring ideas and information)
- Noticing (registering ideas and information happened upon fortuitously)
- Describing (an early stage in understanding new information)
- Distilling and encapsulating
- Interconnecting
- Deducing
- Imagining (includes obtaining insights and conjectures)
- Judging/Evaluating

*Interrogation* includes reading relevant materials and the questioning of oneself or others. We use it: to gather meaning; when critiquing ideas and information (“Does this make sense?”); and exploring possibilities. We *notice* variation, features of data (e.g. patterns and exceptions), little pieces of information or ideas that we come across by happenstance that may have a bearing on our problem. *Description* is an early stage in getting to grips with information. *Distilling and encapsulating* apply to both ideas and information. We try to extract essence from surrounding detail. *Interconnecting* consists of finding relationships between ideas and enlarging our mental models to encompass these interrelationships. It is the basis of understanding and the springboard for insight and conjecture, both of which we have filed under *imagination*. Important uses of imagination include forming mental pictures (models) of a situation when coming to grips with a problem, imagining what its dynamics might be and conjecturing causes for patterns in variation. *Deduction* allows us to explore the implications of our models and make predictions. We apply *judgement* to such things as the reliability of information, the usefulness of ideas, the practicality of plans, the “rightness” of encapsulation and all of the other decisions involved in model building.

There is a continual and complex shuttling going on between these modes of thinking at all stages of statistical cycle. Which modes are involved depend upon such things as whether one is currently trying to extend one’s understanding, or to critique, or to generate new ideas for future use.

#### ATTITUDES OR DISPOSITIONS

Entry into thinking modes is affected by the following personal attributes which we have characterised as attitudes or dispositions: curiosity; scepticism; awareness; openness; a propensity to seek deeper meaning; and engagement. Curiosity prompts us to want to ask questions, to want to find out. Scepticism leads us to critique information, ideas and plans. Awareness helps us notice information and ideas pertinent to the problem when we happen upon them. Openness helps us to register and consider new ideas and information that conflict with our own assumptions. By a propensity to seek deeper meaning we mean not taking things at face value and being prepared to dig deeper. Finally, engagement refers to having some personal connection with the problem. In our experience, curiosity, awareness and the propensity to dig deeper automatically become intensified whenever we find a problem compellingly interesting or important.

One might ask whether these “attitudes” are innate or can be taught. We argue elsewhere that a great deal can be done for specific circumstances by ingraining appropriate “worry questions” and “trigger questions”.

## CONSTRAINTS

Many things constrain the depth and quality of thinking

- Knowledge and preconceptions
- Other personal — Low levels of ability in the “thinking modes” above; low levels of the “attitudes” above; poor communication skills; a lack of perseverance or confidence
- Environmental —time, money and equipment; constraints imposed by clients

Successful problem solving involves bringing everything relevant that you know to bear on the problem, and often much of what you merely suspect. Ignorance is obviously a constraint. Unfortunately, much of what we think we know is not soundly based. The preconceptions, or false knowledge, can lead us astray in many ways by, for example, blinding us to possibilities and desensitising us to important information. This does not only happen at a personal level. One of the most time-honoured methods of approaching problems is to look for a similar problem and imitate the methodology. We follow “precedents”. In applied research, this happens all the time and often the statistical methods of the precedents are inadequate. This brings us naturally into constraints imposed by clients. These run deeper than time-and-materials constraints. The problem territory tends to be mapped out and ring-fenced by the client. The client is often the chief source of context information so the statistician is not only constrained by the quality of communication and the extent of the client’s knowledge, but will also tend to take on board the client’s preconceptions. As the client is the final arbiter, the statistician is constrained by what the client can understand and accept. This can be strongly influenced by “precedents” and a number of psychological factors (discussed elsewhere). Real statistics is less about the pursuit of the “correct” answer in some idealistic sense than about doing the best one can within constraints.

## TECHNIQUES

The following elements are techniques used by thinkers:

- Using past experience with “similar” problems (“following precedents”)
- Modelling and analytical toolboxes (e.g. process/system models, problem solving models, statistical models and tools of design and analysis)

“Toolboxes” is perhaps rather derogatory here. The conceptual dimensions of these systems also enrich the range of models that we can conceive of, thus helping us to understand the context.

### CONTRIBUTIONS FROM THE QUALITY MOVEMENT

The quality movement’s definition of statistical thinking starts from the omnipresence of variation, something that statisticians we interviewed from other areas recognised, but only when prompted. Other areas of statistics failed to appreciate and enunciate variation as a unifying principle for statistics. However, the theme has been picked up from the quality area and is now gaining wider currency in statistics education (see Moore 1997). The quality definition also stresses the process model —the viewing of activities as interconnected processes. Each step of the process can itself be analysed, at the next level of detail, in terms of processes. All of the statisticians we interviewed stressed the critical importance to their work of “getting to grips with the problem.” The process model and the associated diagrammatic tools (flow diagrams, cause-and-effect diagrams, etc) have provided the whole statistical profession with a powerful set of tools for analysing and “getting to grips” with real-world problems. A related tool is Joiner’s 7-Step Process (see Joiner, 1994). This is a problem-solving model aimed at process improvement problems. Polya (1945) describes his well-known model for solving mathematical problems. Many people have found these models useful in finding solutions to process improvement problems and mathematical problems, but we should look beyond the specifics. The simple principle underlying these models is, *systemise what you can and stimulate what you cannot* (i.e. use trigger questions to stimulate thinking along desired directions). This combining of process models and trigger questions provides a modelling framework for improving higher-level statistical thinking of enormous untapped potential. The last element of the quality movement’s definition of statistical thinking highlights some of the ways in which statisticians deal with variation and addresses the role of variation in the particular context of process improvement.

At a more detailed level the quality movement's brainstorming methods are a significant contribution to thinking. A two-stage process is taught. First comes an idea-generation phase. This is followed by a criticism and evaluations phase. Criticism is forbidden in the idea-generation phase, in recognition of the fact that criticism and the urge to censor can stifle the generation of new ideas and erode the confidence to suggest them. Although brainstorming is taught for groups, the process works for a single individual as well. This is just one of several methods taught by the quality movement for generating and organising ideas in a group setting.

## DISCUSSION

This discussion has concentrated mainly on broad aspects of thinking and has only pointed to the inherently statistical aspects. This has been dictated by space constraints. In a much longer paper currently being written, we begin with an expanded version of the ideas presented here and then attack the statistical specialisations from two directions. First we attempt to characterise statistical approaches to variation, the first of the "foundations of statistical thinking" listed above. Second, we go through the statistical cycle (Problem, Plan, Data, Analysis, Conclusions) and highlight some important aspects of statistical thinking at each stage.

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