IS THIS GAME FAIR? THE EMERGENCE OF STATISTICAL REASONING IN YOUNG CHILDREN

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Cross cultural investigations were conducted about the representation and development of statistical ideas as students, from three countries, engaged in building sample spaces while playing dice games. The studies took place in Brazil, Israel, and the United States. At each site, small groups of students were videotaped in a regular classroom environment while working with a partner. Student ideas about chance, sampling, sample space, probability, and fairness are described. Reports include student theories about the fairness of the games and what students regarded as evidence for their theories.

INTRODUCTION

By observing students engaged in problematic tasks that are designed to stimulate thinking about particular ideas, it is possible, often, to gain knowledge about student thinking and to make inferences about students' developing representations. The use of videotaping as a tool provides for even a finer analysis of the development of ideas in students and makes possible the tracing of the origin of their ideas. Videotape records provide a rich data base that can be revisited and studied in detail as new theories are posed and as new questions arise. The videotape recordings make possible the study of individual student cognitive growth in a classroom social setting (Davis, Maher, and Martino, 1992). Hence, classroom research using videotapes makes possible a detailed examination of the development of mathematical ideas in students. Analyses of these tapes along with students' written work and researcher notes allow consideration of a wide range of cognitive and affective aspects that come into play in studying classroom learning and teaching.

The cross cultural studies reported in the collection of papers from the session on children's early statistical reasoning come from a study of videotape data collected at three sites. The students range in grade from elementary to college level. All were challenged with the same two problem tasks,[1] A Game for Two Players and Another Game for Two Players. Both tasks were developed as part of a longitudinal study of children's thinking for the strand of probabilistic and statistical thinking (Maher, 1995; Dann, Pantozzi, and Steencken, 1995; and (Maher, Martino, and Pantozzi, 1995).

BACKGROUND

At the Rutgers site in the United States, a longitudinal study[2] of the development of mathematical ideas in children has made possible the videotaping of the same children doing mathematics for several years. The cross cultural work was motivated, at least in part, by sharing earlier work from a longitudinal study of the development of probabilistic thinking in children. Portions of this work were reported at ICOTS 4 in Beijing, China (Maher, 1995). Observations came from a class of sixth-grade children who worked on a set of problem tasks designed to engage them in discussion about ideas of fairness.

THEORETICAL PERSPECTIVE

Videotaping students engaged in problematic tasks makes it possible to gain knowledge about their thinking and to make inferences about their developing mental representations. These data provide for close observation of students who, while thinking about a particular idea, cycle through a series of steps to build a representation of the knowledge that they regard as relevant to the situation. As they check out their ideas, new evidence may emerge that can lead to a re-evaluation of the original representation. In turn, this may result in a rejection or modification of it, or, perhaps, further evidence to support it. The sequence may involve several cycles before a satisfactory representation is built (see, for example, Davis, 1984; Davis and Maher, 1990; Davis, Maher, and Martino, 1992).

Longitudinal research of the same students makes possible the tracing of the origin of particular ideas. It enables, also, further study of the conditions under which ideas are retrieved, modified and/or extended over time. The establishment of a data base that can be studied in detail over time makes possible an examination of the socio-cultural origins of particular ideas in the classroom community.

THE RUTGERS STUDY

In the Rutgers study, classroom conditions were established so that the students could work on a series of problematic tasks over a few days for extended class periods (approximately one and one-half hours in duration). The students were seated around three tables as three groups of five each. For this report, three cameras (one at each of three tables) were used to videotape student problem solving over a two-day period (Friday and Monday).

Data Collection

The games were given, one at a time, to the entire class. Students were instructed to pose hypotheses about whether or not the game was fair and to explain their reasoning. They were then asked to play the game with a partner at least ten times. Teams worked at their own pace and without teacher intervention. Following the paired problem solving, students shared their work and compared their solutions with one another. Using overhead transparencies, they worked to prepare a presentation of their findings for whole class consideration.

Guiding Questions

Several questions guided the study of the video tape data.

(1) What representations did students build to describe the problem space?

(2) How did their representations help or hinder them in reasoning from the data?

(3) How did their representations change over time?

(4) What factors (e.g., conversations among students, interactions with the teacher; information from playing the game; building models) contributed to changes?

Day 1: For the first game, all of the students almost immediately asserted that it was unfair. The groups proceeded to modify the game in various ways so that each player would have the same number of opportunities for a score.

When the second game was presented, there were variations in students' initial responses. Several students claimed that Player A had the advantage, since there were 6 opportunities for a point (as compared to 5 for Player B). Others suggested that this game was "probably fair", since Player B had the advantage of certain numbers that were easier to roll, thus, compensating for Player A's additional number. A number of ideas were offered concerning which sums were more likely to occur. Individual students asserted that "even" sums were harder to roll than "odd" and that "higher" sums were easier to get than "lower" sums.

One student, Jeff, remarked to his classmate, Romina, that "snake eyes" (two) and "box cars" (twelve) were the most difficult sums to roll and that seven was the easiest. The students in each group played the game several times, recording their scores. In some cases, student data indicated that Player B was by far the more frequent winner. This led some students to revise their previous hypotheses. At the close of the session, the students were asked to think about the game, to play it as many times as they liked, and to return to the second session with any results, hypotheses, and/or explanations that they could develop.

Day 2: Students arrived in class and began immediately to talk about the game. All agreed that the game was unfair. Stephanie came to class with charts that provided elegant graphic organizations of the 36 possible outcomes. Students from each group were instructed to share their ideas and to develop a modified game that they could defend as “fair”. Stephanie, was asked by Ankur to share her graphic displays. Several students gathered to hear her explanation. Meanwhile, the two other groups continued their efforts to make the game fair. In each case, they reasoned from a sample space of twenty-one outcomes.

When encouraged to talk about their findings in a whole class discussion, students argued about what constituted an outcome. The conversation that ensued follows:

The Class Discussion

Teacher:	Do you think you could give some insight into why B has an advantage and even more so what kind of an advantage B has?
Amy:	Well I think that B has the advantage because he has like the numbers that a lot of people get like if they’re playing a dice game. They usually get those kind of numbers instead of like a 12 or an 11 they usually get 7’s or 6’s or 8’s, 9’s.
Teacher:	Why’s that?
Amy:	Because they have... like they have different pairs that can add up to the numbers...like 6, 3 and 3 or 4 and 2.
Teacher:	OK. So you’re telling me there are two ways you can get 6, 3 and 3 and 4 and 2. So is that what you’re telling me?
Amy:	Yeah.
Teacher:	Well 3 you can get how many ways?
Amy:	One.
Teacher:	OK. So Amy is telling me there is one way you can get three. And what’s that way?
Amy:	2 and 1.
Teacher:	Amy says you can get three by 2 and 1...one way...and she can get six by?
Jeff:	2 and 4, 3 and 3, 5 and 1.
Teacher:	Three ways. Do you all agree with that?
Ankur:	No.
Teacher:	Ankur, Ankur doesn’t agree with that.
Ankur:	I say for three, there’s 2 and 1 and 1 and 2, because 2 is on one die and 2 is on the other die and 1 is on the one die and 1 is also on the other die. You

	can have 2 on this die and 1 on this die...or you can have 1 on this die, whatever it is, 2 on this die.
Someone:	Two different combinations.
Michelle	We are working with sums.

Michelle continues to explain her idea about working with sums. Meanwhile, Stephanie moves to the next table and shares her graphs with Jeff.

Teacher:	OK. So we have some disagreement here. Can somebody tell me what the disagreement is? Who can summarize what the disagreement is? Michelle?
Michelle :	He's saying that you have 1 on one die and 2 on the other. But you can also have 2 on one die and 1 on the other. But it is the same thing. We're working with what it equals up to not the numbers that are on the die. We're working with what it equals, not what...

Michelle is then interrupted by Jeff who explains why Ankur's explanation made sense.

Jeff:	Unfortunately he makes somewhat sense because actually you do have two chances of hitting it.
Stephanie:	What?
Jeff:	See, look. Because, if you roll...if this die might show a 1 and this might show a 2...but next time you roll it might be the other way around.

Stephanie, standing at the overhead projector, was preparing to share her two representations that demonstrated a sample space of 36 outcomes to the entire class. Jeff continued to explain his new understanding and decided to discard the earlier model developed by him and his group that was based on a sample space of 21 outcomes.

Jeff:	And that makes it two chances to hit that even though it's the same number. It's two separate things on two different dies.
Stephanie:	Therefore, there's more of a chance. Therefore, there are two different ways. Therefore, there are two ways to get 3.
Jeff:	And that throws a monkey wrench...and that just screws up everything we just sort of worked on for about the past hour.

Stephanie, satisfied with Jeff's explanation, presented her charts to the class. Notice, in reporting, that she changes her initial reference of "my" chart to "our" chart.

Stephanie:	OK. What this is... this is my chart...our chart. What this is over ere...these numbers [pointing to numbers along the vertical line of her histogram] these are how many times that these numbers [pointing to numbers along the horizontal line of her histogram] these are the numbers you can roll...come up. All right. And 2 comes up once and...and I'll show you this

	part in a minute because that'll explain it easier [referring to her chart that shows the sums].
Jeff:	Couldn't 2 come up twice then?
Ankur:	No, because Jeff,...1 on one die and 1 on the other die is still the same thing.
Jeff:	OK.
Stephanie:	All right.
Teacher:	Do you understand that Jeff?
Jeff:	Yeah, because it just seems like even if you do switch them you ...it'll still be like the same thing.
Stephanie:	All right.
Michelle:	But is that the same thing as that? [illustrating (1,1) with the dice]
Jeff:	No, but this...look on this one. You have 2 and 1, but on this, it doesn't matter. You can just...you know what I'm trying to say here?
Matt:	God, it makes sense.

CONCLUSIONS AND IMPLICATIONS

The videotapes show the students talking about their ideas, listening to one another, and explaining their thinking. When the input data changed (as in the case of Jeff), ideas were modified. The teacher did not tell the students how to think about the problem; nor did she show them how to decide on what constituted an outcome. The students dealt with the conflict, evaluated new evidence, and relied on what made sense. Their written work and conversations indicated that they were developing a new understanding of the problem and that they were building up ideas in their own minds by assembling experiential components that were learned earlier.

The episodes from this particular classroom research show that social influences are essential to student learning. These influences, as indicated in the accompanying papers, suggest that classrooms that are organized for students to work and think together, re-examine existing ideas, and explore alternative ones provide the setting for investigating and building basic mathematical ideas.

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Endnotes

[1]The problem tasks are as follows:

Game 1, A Game for Two Players: Roll one die. If the die lands on 1, 2, 3 or 4, Player A gets 1 point (and Player B gets 0). If the die lands on 5 or 6, Player B gets 1 point (and Player A gets 0). Continue rolling the die. The first player to get 10 points is the winner. Is this a fair game? Why or why not?

Game 2, Another Game for Two Players: Roll two dice. If the sum of the two is 2, 3, 4, 10, 11, or 12, Player A gets 1 point (and Player B gets 0). If the sum is 5, 6, 7, 8, or 9, Player B gets 1 point (and Player A gets 0). Continue rolling the dice. The first player to get 10 points is the winner. Is this a fair game? Why or why not?

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