Teaching Econometric Theory from the Coordinate-Free Viewpoint

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1. Introduction

The principal aim of this paper is to demonstrate how the coordinate-free methods of linear statistical models may be adapted to the analysis of econometric models, and to explain why such methods are useful for teaching purposes.

The application of coordinate-free methods to linear statistical models cast in Euclidean space was first introduced by Kruskal (1961). Later, the methodology was extended to multivariate analysis by Dempster (1968) and Eaton (1970). Drygas (1970) and Philoche (1971) give coordinate-free expositions of minimum-variance linear unbiased [or Gauss-Markov (GM)] estimation, and Seber (1964a,b,c and 1980) makes use of coordinate-free methods in a comprehensive treatment of the linear hypothesis.

2. Coordinate-free methods

A coordinate-free argument is an argument that does not depend on a specific set of coordinates. Consider, for example, the n×k real matrix X of rank k. The set of all linear combinations of the columns of X forms a vector subspace called the range-space of X; this is written $\mathcal{R}[X] = \{\theta\colon\ \theta = X\lambda, \lambda \in \mathbb{R}^k\}$. The dimension of $\mathcal{R}[X]$ is the rank of X, and since X has n rows, $\mathcal{R}[X]$ is a subspace of $\mathbb{R}^n$. If Z = XM, M being a k×k non-singular real matrix, then Z has rank k and $\mathcal{R}[X] = \mathcal{R}[Z]$. Clearly there are many more matrices like Z which may be constructed as k linearly independent linear combinations of the columns of X, and each will generate the same subspace. Let this subspace be $\mathcal{X}$; then $\mathcal{X}$ has dimension k and the matrices X and Z are examples of different bases of $\mathcal{X}$ in $\mathbb{R}^n$. If $x = X\alpha$, then α is said to be the coordinate vector of x relative to the basis X of $\mathcal{X}$; if $x = Z\lambda$, then λ is the coordinate vector of x relative to the basis Z of $\mathcal{X}$. If a basis is not specified, then the simple notation $x \in \mathcal{X}$ suffices,