The Gravity Feed Model of Teaching - Laws and Strategies

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1. Introduction : Laws

There are laws and laws. I shall not discuss those promulgated by parliaments and dealt out by courts. Nor do I wish to talk of the Law of Gravity or the Law of Averages. The laws I shall discuss are really sweeping generalisations about the state of things. An example from the world of science is Newton's First Law, which states that "Every body continues in a state of rest or uniform motion in a straight line unless acted on by an external force." This law embodies what, in Newton's time, was a startling insight - that uniform motion in a straight line is in some sense equivalent to standing still. It inspired a great deal of new thinking in science and mathematics. More subtle descriptions of motion have been developed as a result of this thinking, but the original law retains its ability to provoke thought.

Among laws about human behaviour Parkinson's Law is deservedly pre-eminent. This famous law states that "Work expands to fill the time available for its completion." Like Newton's Law, it is neither verifiable nor falsifiable, and therefore not amenable to statistical investigation. But after only a small amount of reflection on our own experience we see that it is true, and very useful.

The laws I shall introduce share some of the characteristics of these two laws, and help to throw light on a very widely-used approach to teaching statistics in tertiary institutions.

2. Mainstream

Tertiary statistics courses are commonly classified as either "service" or "mainstream". This is slightly confusing because in the main "service" courses are taken by far greater numbers of students. "Mainstream" courses are those which deal with ideas fundamental to the discipline of statistics and which are designed to educate the leading statisticians of the future - such as the method of maximum likelihood, moment generating and characteristic functions, consistent estimators, sufficient statistics, convergence in probability and almost sure convergence. The context of this discussion is mainstream courses, though the main ideas apply equally to service courses.

An assumption often made about students in mainstream courses is that they are competent and independent learners - that they have developed study strategies which will be effective no matter what teaching strategies come their way. In the course of 12 years' experience as an academic adviser in mathematics and statistics, I have found that a large percentage of students do not have the skills to become independent. Many are continually anxious about the possibility that their statistics courses will get too hard for them and that their inadequacies as students/statisticians will be exposed. Others, wiser in the ways of the world, realise that this will probably never happen and that they can get by with very little understanding. Students in both of these categories tend to become "minimalist" learners. They learn to beat the system by doing just what is necessary to pass (or to do well) in examinations. One student said to me "I can pass the exam, but next week I won't have any idea of the concepts. Next semester's courses will be assuming things I don't know."

So, many students are not building for themselves, as we hope they do, a coherent set of concepts in statistics. Instead, they avoid the effort required for such learning and develop in the process what seems a rather cynical attitude. The students themselves don't see it as cynical. They are inclined to describe their minimalist approach as a realistic response to the difficult situation in which they find themselves. And indeed this approach is very rational and pays dividends in the short term, which is the only term that counts when people are under pressure.

3. The Gravity Feed model

The most extensively used teaching strategy is *explanation*, in lectures and tutorials, of fairly long sequences of reasoning. Because of pressure to "cover" large amounts of material, there is rarely much opportunity for discussion or questions. In tutorials, where students are in smaller groups, discussion and questions often do not eventuate, and tutors feel compelled to fill the resulting gap with more explanation. The students come away with extensive notes, but mostly they have done very little thinking during the "contact" time with their teacher. And they have a problem - how should they make use of the notes?

I call this reliance on explanation the Gravity Feed (GF) model of teaching, because its use seems to imply that knowledge can be thought of as a liquid commodity which flows under gravity. In the GF model, teaching is nothing but arranging for efficient delivery of the commodity. After delivery, it seems to be assumed, the commodity is available to be used by its new owners.

The GF model really is as simple as the diagram in Figure 1 implies. The lecturer's simple-minded belief is: "If I have explained it then the students know it". When, in the end, events force a retreat from this position, extensions to the GF model offer a way out in the form of the GF model of the student, illustrated in Figure 2.

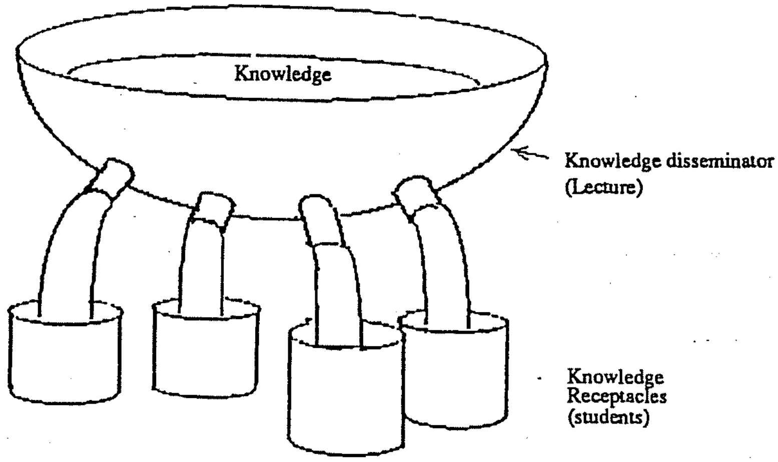

FIGURE 1
The Gravity Feed model of teaching

This model of the student allows a variety of explanations of a student's failure to be able to use delivered knowledge. Learning is conceived of simply as storage (or memory) of knowledge, and the most widely-used explanation of failure is in terms of lack of storage capacity - students are simply unable to store the amount of knowledge delivered. There are, however, many other possible explanations, some of which are suggested in Figure 2.

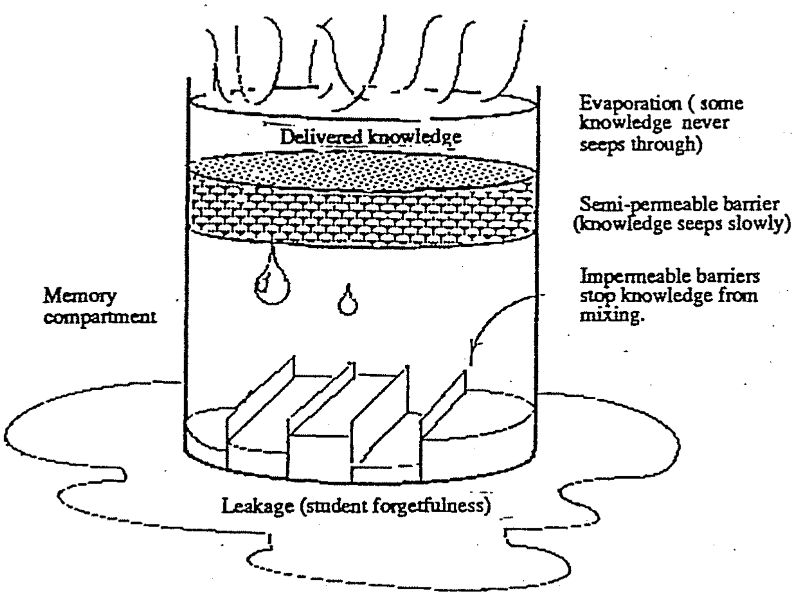

FIGURE 2
The Gravity Feed model of the student

3.1 *Bock's Law*

One problem for the otherwise attractive GF model of teaching as explanation is that it runs counter to Bock's Law (due to Hanne Bock, formerly of La Trobe University, Melbourne). Bock's Law, for any explanation, is that "If it can be misunderstood, it will be, and if it can't be misunderstood, it will be." A corollary is that no explanation will be understood by all who hear it, and with a few simple regularity conditions (maximum attention span of two minutes, randomly arriving distractions, equipment failures) one can deduce that hardly anyone will understand any explanation, no matter how clear. The consequence for the GF model is that the semi-permeable barrier in the model of the student is all but impermeable.

It is easy to find empirical evidence of the action of Bock's Law, though such evidence does not seem to be well documented. In my experience and in the experience of students as reported to me, statistics (and mathematics) tutorials (small group teaching sessions) are characterised by a very low level of active student questioning. No-one believes that this is due to a generally high level of student knowledge. Most will acknowledge that there are powerful reasons associated with classroom politics which can account for a certain amount of diffidence.

But these political reasons do not seem to me to be the whole explanation. I am convinced that there is a deeper reason. This is that, for the most part, students don't ask questions because they have no idea what questions to ask. This in turn is due to the fact that the lectures they have attended have "gone right over their heads". Their lack of understanding inclines them not only not to ask questions, but also not to make much effort to understand until examinations loom. And when examinations do loom they direct their efforts to copying and learning model answers, which is a very effective strategy for passing examinations. Such learning as this seems to be stored in very short-term memory. And so it comes to pass that many students succeed in statistics examinations without achieving a lasting understanding of statistical concepts.

Given that observations such as these are surely not startlingly new, what charm accounts for the continuing overwhelming popularity of the GF model?

3.2 *Taffe's Law*

The GF model has been running for centuries in most Eastern and Western cultures, so that like the body of Newton's Law it requires some external force to change its course. Its charm appears to be that it is very cheap to run when compared with most strategies based on active involvement of students. We have to conclude that it is here to stay for some considerable time.

But consider for a moment the unlikely scenario of cost being no object. I maintain that even in this case the GF model will have a black hole-like attraction. This is a consequence of Taffe's Law, which states that "In human communication the default strategy of all parties is to minimise input." Why, you may ask, do we try to minimise our input into a communication? Because this has the effect of restricting the power of the maximum possible feedback from the communication. (Game theorists will recognise the well-known "minimax" strategy.) Why are we concerned to minimise feedback? Because coping with feedback drains our emotional energy. Taffe's Law is also known as the Law of Conservation of Emotional Energy.

Applying Taffe's Law to the problem of *teaching statistics in a tertiary setting*, we see that the GF model is a very good solution. By keeping students in the dark it minimises the chance that they will apply an external force (by asking questions) to slow down or derail the progress of presented material. But, you will ask, does it achieve this by minimising input? To see that it does we need only compare the work involved in preparing a seamless web of reasoned argument with that of making contingency plans to respond to the unpredictable results of student activities.

From the student's point of view also the GF model gives a close approach to minimal input and feedback. Transcription requires only small amounts of energy and is a distraction from the responsibility of thinking about the material. The better organised the material is the less reason there is to disturb the flow by asking questions. In a well presented lecture there is no occasion for the student to receive any feedback at all from the lecturer.

4. The character of explanation

The person giving an explanation is the one who learns a great deal from it. The preparation has involved reorganising the elements of the problem into some congenial shape and considering the problem from the imagined point of view of the audience. These are instructive processes. But the most instructive process is the actual manufacture of the argument - the inventing of words and illustrations to encapsulate it. This is the part of the process that increases the author's control over the argument.

Figure 3 shows how the benefits of explanation are distributed - the lion's share to the explainer and very little to the explainee.

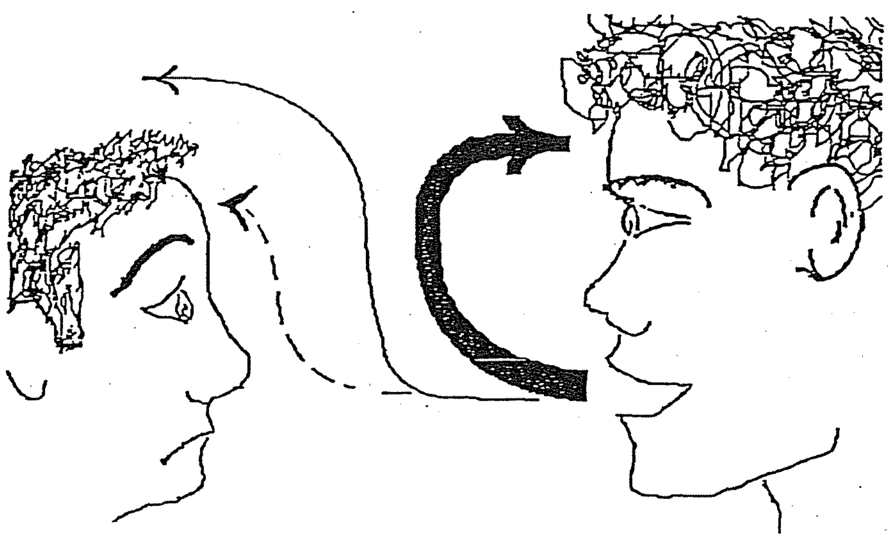

FIGURE 3
The benefits of explanation

When the explanation is given again and again, as sometimes happens, it tends to become finely honed and pared back so that only the "essentials" remain and the argument is expressed tersely and economically. The more refined it becomes, the more educational value is wrung from it by its author and the less accessible it becomes to readers, listeners, and students.

Arguments given in textbooks often bear the stamp of such refinement. Consider the following example, from a well-known text by Hogg and Craig (1970).

"*Example 2.* Let $Y_1 < Y_2 < \dots < Y_n$ denote the order statistics of a random sample $X_1, X_2, \dots, X_n$ from the distribution that has p.d.f.

$$f(x;\theta) = e^{-(x-\theta)}, \quad \theta < x < \infty, \ -\infty < \theta < \infty,$$

$$= 0 \text{ elsewhere.}$$

The p.d.f. of the statistic Y_1 is

$$g_1(y_1;\theta) = ne^{-n(y_1-\theta)}, \quad \theta < y_1 < \infty,$$

$$= 0 \text{ elsewhere.}$$

Thus the joint p.d.f. of $X_1, X_2, \dots, X_n$ may be written

$$e^{-(x_1-\theta)} e^{-(x_2-\theta)} \dots e^{-(x_n-\theta)} = g_1(\min x_i;\theta) \left\{ \frac{\exp(-x_1 - x_2 - \dots - x_n)}{n \exp[-n(\min x_i)]} \right\}.$$

By the Fisher-Neyman criterion, the first order statistic Y_1 is a sufficient statistic for θ."

This argument is only four sentences long. The scheme of these sentences is: framework for discussion; result seemingly pulled from thin air; unlikely-looking consequence; flourish, in which the reason for the whole argument is revealed. Elegance and economy rather than plainness and transparency are the key notes. Such arguments take some deciphering and quite a bit of filling in of detail. Linguistically capable students who are also confident with the relevant statistical theory will be able to pull it apart and piece it together again, with some difficulty. They need not only ability but reliable decoding strategies. Students without such equipment tend to give up in despair, learning in the process the dreadful lessons of failure. In this way a great amount of talent is lost, through lack of useful strategies.

5. Strategies

Secondary school mathematics does not prepare students well for acquiring an understanding of concepts in the environment of the GF model. They learn in secondary school the strategy of putting in many hours completing rafts of "examples", learning by osmosis, skills that are useful in examinations. They tend to see textbooks as mines of such examples which would be next to useless without the answer section at the back: a good textbook is one with few errors among the answers. To many, mathematics is nothing but a series of hurdles - an exercise programme to build up mental fitness, a test of otherwise useless skill. Unfortunately, these attitudes transfer easily to tertiary statistics, where the agenda for many students is learning to recognise problem types and applying appropriate algorithms. In this way it is possible to do quite well without much understanding of statistical reasoning. "In statistics you don't try to understand, you just *do* it", as one student remarked to me.

In the GF model a student's failure to build up an understanding of concepts is a sign of lack of ability (inadequate memory compartment capacity). In reality it is often just an instance of the operation of Taffe's Law. Playing "find the right algorithm" is a low input solution to the problem of negotiating a statistics course - possibly the minimal input solution if we take as a boundary condition that the student passes. A big advantage is that lack of understanding need never be admitted, so that negative feedback can be avoided. The technique is to ask "Is this the right algorithm?" (Is this an independent samples t-test or a matched pairs?) In the student's view this creates a good impression on the teacher, as it shows that only small details are standing in the way of a complete mastery of the subject.

The downside is that by adopting this approach the student closes off paths to understanding which might be opened by questions such as "Why is it distributed that way?" or "How do you know that this follows from that?". In a GF model course there is very little opportunity for such questions to arise spontaneously. The student needs to develop them privately and then create opportunities to ask them. Students whose concept of study in statistics and mathematics is along the lines of "fitness training" are simply not able to develop useful questions, because the idea of doing this has not occurred to them. They need to be taught that there are ways, and then they need lots of encouragement to try them, for the inertia of minimalist practice is great. I describe here two strategies for developing questions that are useful for students in GF courses.

5.1 *Translation*

Much of the raw material for study is in the form of written arguments in lecture notes or texts. A most useful strategy is to try to make a paraphrase or *translation* of such arguments. This reveals points which are not understood and provides a systematic way of generating questions along the lines of "What does this mean?" and "Why does this follow?". Referring to the example I have quoted from Hogg and Craig, a translation of the first sentence might run:

> Suppose $X_1, X_2, \ldots, X_n$ are drawn from an exponential distribution which is positive for $x > \theta$, for some real number θ, and that the Xs are written in ascending order of magnitude and re-labelled $Y_1, Y_2, \ldots, Y_n$.

Reading this translation you can see that it was written by someone who recognises the distribution, understands the restrictions given, and knows what order statistics are. If, for example, the restriction on x had been omitted, you might suspect that its meaning was not understood. Making translations forces you to pay attention to details that you may just glide over if you are reading. For a student, the object of a first translation is to compile a list of questions that need to be answered before a complete translation can be done. The questions may be answered by further reading or by discussion with other students or teachers.

5.2 *Reconstruction*

Translation is a strategy for explication of an argument as given. It enables a student to convert the author's ideas and concepts into his or her own, and also to clarify the logical connections between statements. When you have a satisfactory translation it is time to attempt a *reconstruction.* This means to begin with a blank page and try to rewrite the argument in any acceptable form (original or translation). This is a task which calls on different skills from those employed in translation. Doing a translation does not require you to be able to see the author's plan - the way he has ordered his thoughts and decided how to present them. Doing a reconstruction involves you in planning what comes next and eventually seeing the whole scheme of an argument. What happens in the initial stages is that you find places where you don't know what comes next. When you refer back to a translation of the argument you may not be able to see why the next step was chosen the way it was, in which case you are able to generate a useful question. Or you may suddenly see why it was chosen, in which case you have learned something. With practice you begin to see different ways in which the argument could have been arranged, in which case you are gaining control of the concepts involved. In the end, you may see different arguments leading to the same conclusion, or extensions and variations to the argument.

Both of these strategies are expensive of effort, especially for students who have difficulty imagining that anything other than the strategy of wading through masses of similar examples is possible. Unfortunately, many are so well trained that they cannot imagine it. Some who can imagine it see that it runs counter to Taffe's Law and so are not prepared to invest effort on something they can see is not strictly necessary in order to pass examinations.

So to many students translation and reconstruction are hard to sell. But many others have found that the investment is worth the effort.

Reference

Hogg, R V and Craig, A T (1970) *Introduction to Mathematical Statistics.* New York.