Using Real Applications in the Teaching of Probability Theory

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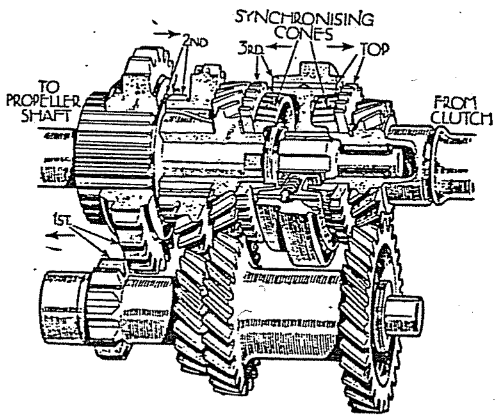
1. The aims of working on real applications with secondary students

Students should gain an insight into the usefulness of what they have learned in class such as the applicability of formulae, ideas, and methods. They should also have the opportunity of developing hypotheses and proving theorems. Probability theory and statistics provide many suitable ways for learning about these concepts. Students will be motivated to learn mathematics by using it to solve real problems. Consider, for example, a discussion of the risks associated with nuclear reactors - we should all be able to critically evaluate official statements and the arguments of so-called experts which are based on apparently legitimate mathematical methods. Looking at real applications in mathematics lessons, students get an insight into the part mathematical sciences play in the real world. We shall give three examples to illustrate our ideas.

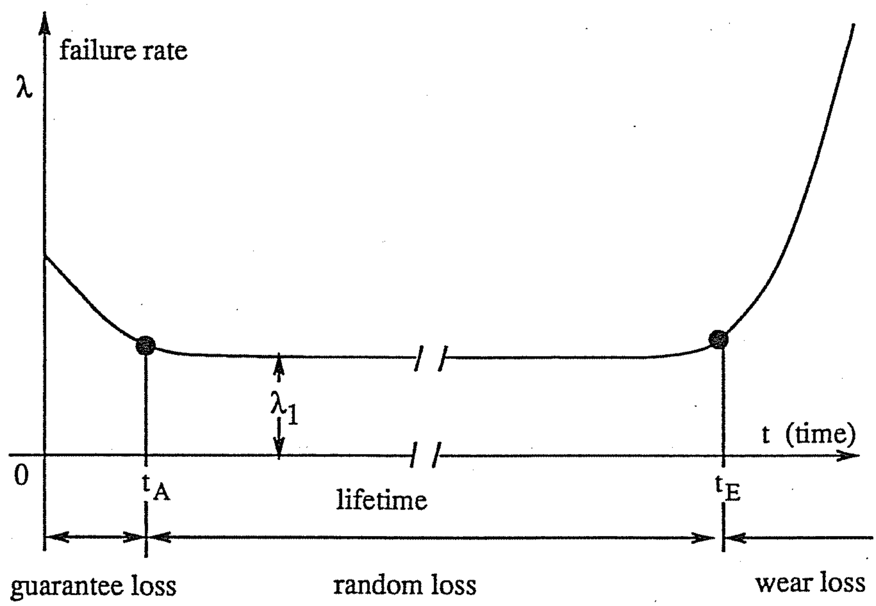
2. The gearbox of a motorcar and the Gaussian distribution

Student teachers discovered this real application of the Gaussian distribution in the mathematical department of the Ford works in Cologne. A gearbox consists of various components such as cogs, clutches, and springs. To construct a gear we put the pieces together on two axes and mount them in the gearbox. On a blueprint you find the nominal measurements of the components but in reality they are not exact. The size of each piece is only approximately that required. Random samples of the cogs, for example, give us measurements which are discovered to fit a Gaussian distribution. Due to the stochastic independence of the different components we see that the sum of their sizes is also Gaussian distributed. Knowing this we can give the manufacturer the measurements which will allow most of the prefabricated parts to fit the gearbox. Finally, we are able to develop special graph paper for the Gaussian distribution (normal probability graph paper) which has one axis scaled to a Gaussian function and the other linear. This can be used to test data which is expected to be normally distributed. Such data is linear on the graph and its mean value can be easily determined.



3. The bath-tub curve or the Weibull-distribution

In 1949 W Weibull, a Swedish engineer, developed a method to test machine components. Since 1970 this method has been used to judge the state of car components by nearly all large motorcar manufacturers (Daimler-Benz, VW, Opel, Ford, ...). Look at the following curve:



This "bath-tub" curve describes the failure rate, λ, of some car component such as headlight, indicator, clutch, tyre, etc. Early on in its life: $t \in (0,t_A)$, the component is under warranty and we have *guarantee-loss* with costs for the producer; this part of the curve should decrease rapidly. During the normal lifetime of the component (typically some years): $t \in (t_A,t_E)$, we have the *random loss*, when the failure rate should be small, nearly constant, and long-lasting if the car is to have a good reputation. At the end of this time the failure rate may increase and repair costs will be paid by the owner. We call this the *wear-loss* period. Most machines and components behave like this; if not they would be replaced by better ones.

Looking for a suitable function to describe the curve we find $\lambda(t) = b.t^{b-1}$, $b > 0$. On the other hand, λ is defind by $\lambda(t) = -(dS(t)/dt).(1/S(t))$; $S(t) = 1-F(t)$. By integration we obtain the Weibull distribution: $F(t) = 1 - \exp(-(t/T)^b)$ with two parameters b and T.

The problem now is to develop a testing procedure for repair data of all service facilities throughout the country to be able to judge the performance of some component. From the above distribution, F(t), using natural logarithms twice, we obtain a probability graph paper for the Weibull distribution. If data chosen at random is Weibull-distributed it forms a straight line on such a graph and the parameters b and T can be readily obtained.

If the above procedure is followed a critical evaluation of the data may be made and appropriate advice given to the manufacturer.

4. The risks associated with nuclear reactors

In Germany we have a publication *Deutsche Risikostudie Kernkraftwerke* (TUV-Verlag, Köln, 1980) to which most experts and politicians refer when they discuss the dangers of nuclear reactors. I developed a project with the aim of teaching school students (aged 17-18) how to critically evaluate statements made in the publication.

First, we visited the research institute for nuclear safety in Aachen. They gave us very good information about safety procedures and the workings of a hot water reactor which is also included in the *Risikostudie*. In the follow-up lessons the students studied some of the chapters in the publication and were surprised to find that they understood most of the calculations and estimations using their knowledge of elementary probability theory. Only occasionally were they forced to learn more mathematics to be able to understand the material. We concentrated our study on one possible accident: the failure of the main cooling system which causes an explosion of the safety box. We considered, according to the data given in *Risikostudie*, only deaths by leukemia and carcinoma caused by radioactive gas.

Our discussions included: What do you think about the interpretations from very small probabilities given in *Risikostudie*? Is it valid to compare risks from nuclear reactors (given as 10.1 deaths per annum) with those from traffic accidents (14,400 deaths per annum)? What do we know about the input data, that is, the mean failure rates of some component? How are these mean values obtained? Is there any account taken of the propagation of measurement errors while computing the risk? Risk means the product of probability rate and damage. Is it correct to include only deaths? Is there no other damage? Finally, what can be said about the mean value of an accident given as E(U) = 1/10,000 per annum? We have to remember that "per annum" means working years of the reactor and not calendar years. World-wide we have had about 5,000 working years of reactor life! On the other hand E(U) is the mean value for one reactor while world-wide there are 400!

In addition, we studied the binomial distribution to describe the behaviour of switches, motors, and so on; the Poisson approximation in relation to the failure of electrical parts; the exponential distribution for the reliability of pumps, valves, and motor components; and finally the logarithmic Gaussian distribution for the mean value of the failure rate of some component.

Altogether I think this was a most interesting course. It was certainly the most popular course when the Chernobyl accident occurred!

Reference

Bungartz, P *Risiko Kernkraftwerke*. Reprint. Mathematisches Institut der Universität Bonn, Wegelerstr. 10, D-5300 Bonn, FRG.