APPROPRIATE TEACHING AND LEARNING OF STOCHASTICS IN THE MIDDLE GRADES (5 - 10)

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Introduction

We will present and discuss the outlines of a curriculum for stochastics instruction in the middle grades. Why such a construction at a time where plenty of analytic investigations are done and are to be done in this domain?[2]

- There is already stochastics instruction and there are many students and teachers who need and wait for better syllabi.

- It see[…] me (in regard of the published studies and from own experiences […]ifferent types of Germany School for more than 10 years) that we h[…]lready many facts which prescribe certain features of a meaningfu[…]ning sequence.

- Such […]uence would serve as a favourable basis for further investigations.

- Last but not least: Didactical constructions never can be founded only on analysis, but must include prospective and normative elements.

Of course, this curriculum merely gives a frame. It must be concreted and filled at the front and even be open for alternative approaches.[3]

Criteria

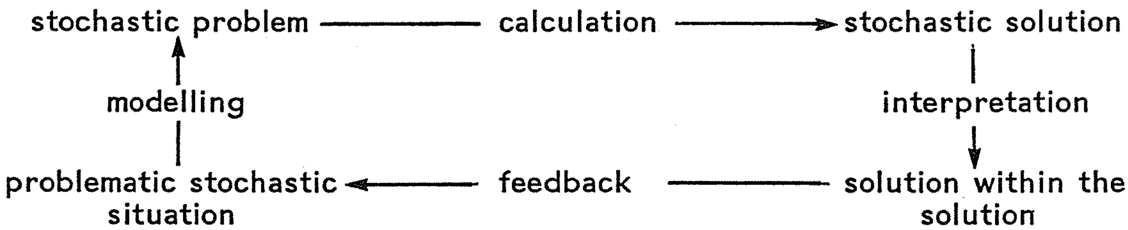
First we will argue four (not independent) criteria for such a sequence.

Structure of the discipline.

This could be misunderstood as elementarisation or even imitation of a systematic university lecture. It means in the contrary that we remember the rise of the discipline and how its methods are used.

Then we see:

- Stochastic work is taking place in a control circuit:



This is a special (and very simplified) version of the general relationship between world (reality) and mathematics (model).[6,7]

- Stochastic work is characterized by a suitable combination of statistical tools (especially within modelling, interpreting and feed-back) and probabilistical methods (calculation).[5]

- Stochastic work has reached and influenced nearly all sectors of daily life and nearly all disciplines.

Student-related proceeding.

That means:

- We have strongly to consider the actual state of our student's cognitive development.

- We have to plan and to warrant a rich arsenal of student activities, especially at the enactive and the iconic stage.

- We have to evoke the primary intuitions of our students for to strengthen the correct ones and to reduce the biased ones (which is a long-time problem.[1,2]

Aim "adequate behaviour in stochastic situations".

- We should create and enhance the ability to pass (as often as necessary) the stations of the control circuit. It is no secret that – up to now – curricula, school books and above all the instruction itself don't sufficiently meet this criterion. Activities within stochastic systems are dominating; modelling, interpreting or feeding back come off badly.

Genetic teaching. That is to say:

- Instead of pushing forward the theory we should arrange our lessons according to the complexity of the situations the students are confronted with.

- As soon as possible we should lead our students through the control circuit and should bring it to their consciousness.

- Game situations are very useful in the beginning of a new phase because they are motivating and easy to model (but should not dominate).

- We should make frequent use of simulations and model experiments (e.g. coin, urn, spinner, Galton board).

- Supporting concepts are not given until they are indeed helpful. This holds especially for concepts out of set theory and combinatorics.

- On the other hand: Chief concepts (such as chance, independence, frequency, probability, expectation, distribution) are introduced as early as possible and successively brought forward.

Learning Sequence

Some remarks:

- The sequence consists of several phases. A phase doesn't cut off but carries on its predecessor.

- Each phase is characterized by its crucial activity.

- To each activity belongs the critical reflexion about it, about its results and about such activities of other people e.g. in the public media.

- The contents of our sequence don't differ too much from those of the official syllabi. This is important for its acceptance by the teachers and a consequence of the experience that improvements are rather due to the change of aims and methods than to new topics.

1. Phase: Experimentating

Contents: simple chance experiments (with one, with several stages (first independent, later on dependent)) from diverse areas (including informative sampling); sampling); collecting, winning, pondering and graphing the results (data); in connection with poly-staged experiments: tree diagram and multiplication principle.

Background: representative confrontation with chance; perception of its frequency and relevance; discussion of the primary intuitions; qualitative approach to:

Central concepts: chance, label, experiment, trial, outcome, (in)dependence, probability.

This phase needs a lot of time and therefore should start as early as possible.

2. Phase: Quantifying

Contents: absolute and relative frequency of outcomes; empirical law of large numbers; probability of an outcome as its expected relative frequency after many trials; frequency and probability distributions (many different types); uniform distribution as a special case and classical probability.

Background: stochastic dialectic: chaos in detail versus order on the whole; construction of a measure for probability which meets this dialectic (and therefore is a priori problematic;[3] introduction to the basic methods of descriptive statistics (and EDA?).

Central concepts: absolute and relative frequency, probability, distribution, uniform distribution.

3. Phase: Calculating

Contents: probabilities in one- and poly-staged experiments; work with the tree diagram, first and second path-rule; simulations.

Background: probabilities by probabilities (especially from the outcomes to the event, from the stages to the whole experiment); passing (and re-passing) the control circuit; advantages and limitations of the probability concept; simulation (with and without computer) – as a control and as an alternative solution.

Central concepts: secondary probability, (in)dependence, simulation.

4. Phase: Characterizing

Contents: location and dispersion parameters of frequency and probability distributions.

Background: power and limitation of these parameters in stochastic situations; advanced work in the control circuit.

Central concepts: mean, median, modus, average deviation; expectation, expected deviation.

With this phase we have reached a first solid level of stochastic abilities and insights. We should try to bring each student to this level even under bad conditions. This is possible as we have shown in a 6 years investigation with more than 1500 students in the German Hauptschule.[4]

5. Phase: Systematizing

Contents: event as set of outcomes; frequency and probability of events; combinatorial rules and its importance for the determination of classical probabilities in case of a big number of outcomes.

Background: resuming, precising and structuring the mathematical part of the stochastic knowledge worked out up to now, motivated by the increasing complexity of the situations; preparation of a further enhance central concepts: event, n-tuplet, permutation, sample, binomial coefficient.

6. Phase: Evaluating

Contents: inverted tree diagrams; conditional probabilities (including product rule, Bayes' Theorem); binomial distribution; simple testing in case of this hypothesis.

Background: evaluating the causes for events which have occurred; evaluating (by sampling) the quality of maintained or supposed probabilities; revising hitherto valid results in the light of new experiences;[2] the problematic nature of such judgements[2].

Central concepts: conditional probability, independence, binomial distribution, sample, test, statistical safety (significance)

Whether the students reach this second, more elaborated level of stochastic reasoning and behaviour or not, at any rate we propose a final project approach to an interesting, important and relatively complicated real-life problem. Such a project meets the above mentioned criteria much better than the normal manner of instruction with its cut off tasks.[2] The students will learn that such a problem matters more than one discipline and that stochastic methods provide only some of the arguments for the final judgement or decision.

In our curriculum for the Hauptschule (4) we have developed the following projects:

- Fill out of ready-made packings

- Heredity of characteristics

- Chances of winning in public games of hazard

- Unemployment

Last remark: Such a rough sequence is a necessary but not sufficient condition for an effective stochastics instruction. What really happens in school depends also (and perhaps more) on the quality and the energy of the teacher.

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