

## ELEMENTARY SCHOOLS STUDENTS' USE OF STRATEGY IN PLAYING MICROCOMPUTER PROBABILITY GAMES

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Games are often used in teaching as a means of introducing and exploring probability concepts, since they provide familiar and practical instances of the notions in question. [For a thorough discussion of the purposes and effects of using games in teaching see Bright, Harvey, and Wheeler (1985).] This paper, however, describes a study (Schroeder, 1983) in which two versions of a game are used as the setting in which students' understanding of probability is assessed. The subjects involved had received no formal instruction in probability prior to the experiment, but during it some of them used intuitions about chance (Fischbein, 1975) as they developed their strategies for playing. There is also evidence that as they responded to the interviewer's questions and explained their strategies, subjects sometimes attended to previously unnoticed features of the situations and developed new strategies as a result.

### Games

The two games used were developed by the investigator and given the names Capture 1 and Capture 2. In the experiment they were played on a microcomputer which was programmed to present the rules, display the gameboard, prompt the moves, simulate the moves of one player, and record each move of each game for later analysis. The rules of the game are given below.

#### Rules for Capture 1

1. Two players play on a number line that goes from 0 to 36. Each player has a mark – either X or O.
2. The players take turns. Each player gets 15 turns. On each turn the Apple will roll two dice marked with the numbers 1 to 6.
3. You can use the numbers on the dice to capture one place on the number line from 0 to 36.
4. You can capture the place that is the sum, or the difference, or the product of the two numbers. You can also use the numbers as tens and ones.

For example, if you rolled 3 and 2, you could capture:

$3 + 2 = 5$  or  
 $3 - 2 = 1$  or  
 $3 \times 2 = 6$  or  
3 tens and 2 ones = 32 or  
2 tens and 3 ones = 23.

5. On each turn you can capture any place that can be made with the dice. You can even capture a place that was captured before by you or by your opponent.
6. The object of the game is to capture as many places as you can. The winner is the player who has the most places at the end of the game. The game is over when both players have taken 15 turns.

### Rules for Capture 2

1. Two players play on a number line that goes from 0 to 36. Each player has a mark – either X or O.
2. The players take turns. Each player gets 15 turns.
3. On each turn the Apple will roll two dice. One is marked with the numbers from 1 to 6. The other is marked with the operations +, -, and x.
4. On each turn you can capture one place on the number line. The place you capture must be made with the number and the operation on the dice and any one-digit number you choose.

Here are some examples:

If you rolled 3 and X you could capture  $3 \times 0 = 0$ , or  $3 \times 1 = 3$ , and so on up to  $3 \times 9 = 27$ .

You could also capture  $0 \times 3 = 0$ , or  $1 \times 3 = 3$ , or  $2 \times 3 = 6$ , and so on.

If you rolled 3 and -, you could capture  $3 - 0 = 3$ , or  $3 - 1 = 2$ , or  $3 - 2 = 1$ , or  $3 - 3 = 0$ .

You could also capture  $9 - 3 = 6$ , or  $8 - 3 = 5$ , or  $7 - 3 = 4$ , and so on.

5. You can capture any place that can be made with the dice. You can even capture a place that was captured before by you or by your opponent.
6. The object of the game is to capture as many places as you can. The winner is the player who has the most places at the end of the game. The game is over when both players have taken 15 turns.

## Strategies

The term "strategy" is used in a number of fields and disciplines including mathematical game theory, artificial intelligence, cognitive (and especially meta-cognitive) psychology, and sports. In the present context, where we are concerned with students, their overt behaviors, and their thinking, we use "strategy" to refer to the algorithms or rules players use to determine which of the available moves to make at any given turn. Modifiers such as "optimal," "novice," "expert," "offensive," "defensive," and others are used with their obvious meanings.

The following four strategies for playing the versions of Capture have been identified:

1. Do not recapture a position you already hold.
2. When possible, capture a position held by your opponent.
3. Capture the position that your opponent is least likely to be able to recapture from you (i.e., the position that can be captured with the fewest dice roll outcomes).
4. Coordinate the three strategies above, giving priority to Strategies 1 and 2, but applying Strategy 3 whenever they do not determine a unique optimal move.

Strategies 1 and 2 may be termed offensive strategies, while 3 is defensive. Strategy 4, the optimal or expert strategy, may be considered a meta-strategy or coordinating strategy. Strategy 3 is the strategy which permits investigation of students' concepts of probability. It is based on the fact that some board positions are "harder to make" than others. The number of dice roll outcomes which permit the capture of each position are shown in Figure 1. The smaller the number of outcomes that permit a capture, the less likely it is that the opponent will be able to recapture that position later in the game.

To assess whether individual students use these or other hypothesized strategies, one might take a behaviorist approach and determine at each move of each game whether the student's play was consistent or inconsistent with the respective strategies. This was done using four variables, S1, S2, S3, and S4, which were defined as having the value +1 when the player's move was consistent with the strategy, having the value -1 when the move was inconsistent with the strategy, and having the value 0 when the strategy did not apply. Unfortunately, the values of these variables are not necessary and sufficient conditions for "having" or "using" the corresponding strategies, because a given behavior may be consistent with a strategy that the student had not thought of, and because computational error or failure to consider all alternatives may result in play that is inconsistent with the student's explicit, intended strategy.

**Figure 1: Number of dice role outcomes which permit the capture of each board position in Capture 1 and Capture 2.**

Capture 1	Capture 2
0 *****	0 *****
1 *****	1 *****
2 *****	2 *****
3 *****	3 *****
4 *****	4 *****
5 *****	5 *****
6 *****	6 *****
7 *****	7 *****
8 *****	8 *****
9 *****	9 *****
10 *****	10 *****
11 ***	11 *****
12 *****	12 *****
13 **	13 ***
14 **	14 ***
15 ****	15 ***
16 ***	16 **
17	17
18 **	18 ***
19	19
20 **	20 **
21 **	21 *
22 *	22
23 **	23
24 ****	24 ***
25 ***	25 *
26 **	26
27	27 *
28	28 *
29	29
30 **	30 **
31 **	31
32 **	32 *
33 **	33
34 **	34
35 **	35 *
36 ***	36 **

A second approach to assessing students' strategies is to ask them open-ended questions such as "Does it matter what move you choose to make?" and "What do you think about when you choose your moves?" It may be argued that the answers to such questions are not valid and reliable indicators of students' use of strategy. For example, they may lead to an underestimate of students' strategies because they interfere with concentration or focus too narrowly on the strategy used in a particular situation rather than on the range of substrategies that an explicit or implicit coordinating strategy might call upon in a different situation. On the other

hand, the questions themselves may enhance students' performance by implying or suggesting ideas that the player would not otherwise have considered or by demanding reflective thinking (i.e., thinking about the player's own thinking). This is particularly likely when follow-up questions are asked in order to clarify a statement, or when the interviewer deliberately asks leading questions.

But by using both these approaches together and recognizing the advantages and limitations of each, conclusions can be reached and the evidence on which they are based can be reported for others to evaluate.

### Experimental Design and Procedures

The subjects in the study were six Canadian children in Grades 4, 5, and 6, ranging in age from 10 to 12 years, representing a range of ability levels. All were familiar with the use of the microcomputer, but had not previously received formal instruction in probability. Each subject was interviewed on three successive school days. On each of the first two days they played Capture 1 twice against a computer-simulated player and discussed the games with the interviewer as they played. On the third day they played and discussed Capture 2 twice. Each interview was audiotaped, and each move of each game was recorded by computer for later analysis. Values of S1 to S4 were computed for each move of each game.

### Findings and Conclusions

Some of the subjects were initially uncertain about the meaning of Rule 5 concerning recaptures, but all but one of them immediately noticed the effect when the computer-simulated opponent recaptured a position they had held. Thus, all subjects discovered, used, and described Strategies 1 and 2. Their comments showed that they thought these strategies were quite simple, even obvious. For example, one student described recapturing a position already held by the player as a "waste of a turn."

Several subjects made the generalization that the larger-numbered board positions are more desirable because they are less likely to be "taken away" by the opponent. Most of these subjects were somewhat tentative in explaining this idea and had difficulty producing exhaustive lists of the ways in which positions could be recaptured. Others, with some prompting, recognized the fact that some board positions changed hands more frequently than others, but could not describe a strategy for minimizing the opponent's opportunities to recapture and did not seem able to relate this, even in informal terms, to probability.

In discussing the games with the interviewer, some subjects seemed to use the probability concepts implicit in the questions later in the discussion. Informal terms (such as "more of a chance" or "a better possibility") were used rather than formal terms. Predictions were used more often than arguments based on probability, but this may have occurred because the questions suggested prediction instead of estimation of likelihood.

In some cases variables S1 and S2 had negative values even after the subject had discovered these strategies; this resulted from computational error, misreading of the number line display, or failure to consider all possibilities. Variables S3 and S4 were sometimes negative for students who seemed to understand these strategies because their estimates of the relative probabilities were incorrect, usually due to failure to consider all possibilities.

Their study demonstrated that at least some students aged 10 to 12 years have ideas about probability that they can use in constructing strategies for these games. Those ideas are probably best characterized as intuitions or "things that one knows without having been taught." The study also suggests that when teachers' questions demand reflect thinking, students may develop more powerful ways of thinking about the situations and construct more expert strategies for playing.

### References

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