CHARACTERIZING THE PROBABILITY PROBLEMS PROPOSED IN THE ENTRANCE TO UNIVERSITY TESTS IN ANDALUCIA

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The aim of this research was to investigate the distribution of the main variables characterizing the probability problems proposed in the university entrance tests in Andalucía (Spain). Specifically, we examined all the problems proposed to the students in the period 2003-2014 (n=144 problems). We considered the following variables: type of experiment and sample space considered, type of probability that should be computed, theorems or properties needed to find the solution, format of data and context. The distribution of these variables serves to obtain conclusions about the problems potential difficulty and the correspondence of these tests with the official curriculum.

INTRODUCTION

The need for probability education has been recognized in the Spanish curricular guidelines, which include probability contents in all the grades since Primary Education. Probability is also included in Mathematics into the specialties of Sciences and Technology, and Humanities and Social Sciences in High School (MEC, 2007; MECD 2015), where we find the following contents:

- Assigning probabilities to events. Prior and posterior probability; joint, conditional and total probability, Bayes' theorem.
- Binomial and Normal distributions. Practical implications of central limit theorem; approximation of binomial distribution by the normal distribution; law of large numbers.

Assessment is an important element that guides teachers and students towards achieving the learning goals. Assessment is given a main role in the transition from high school to university in Spain, since the score the students get in the compulsory entrance to university tests, often determines the student's possibility to follow their preferred university degree (and future career). Paradoxically, probability problems are only included in the tests directed to Humanities and Social Sciences students, although this content is also taught to Science and Technology students.

In this paper we present preliminary results of a project directed to analyse the probability content of the university entrance tests in Spain. The interest of this project is clear, since these tests often influence (more than the curricular guidelines) what is taught to these students, and our results may help the teachers to prepare their students for these tests. These preliminary results consist in the analysis of all the problems posed in Andalucía in the period 2003-2014 and will serve to pilot the method and determine the main variables to continue the analysis of the tests proposed in other Spanish regions and determine possible differences. Below we present the research background and method, as well as main results. Finally, we conclude with some implications for teaching.

BACKGROUND

Theoretical Framework

We base on the onto-semiotic approach to mathematics education (Godino, Batanero, & Font,

2007) where the meaning of mathematics objects is modelled in terms of systems of the practices carried out when solving problem related to that object. In these practices different type of objects intervene; "object" is understood in this theoretical framework in a broad sense and includes problem-situations; language and representations; procedures; concepts; properties and arguments. The mathematical practices can be idiosyncratic of a person (personal meaning) or shared within an institution (institutional meaning).

In mathematics education research we are interested in the meaning of specific objects in teaching institutions; where different types of institutional meanings for a mathematical object (in this case probability) are considered: a) *reference meaning* (global meaning of the probability in statistics), b) *intended meaning;* the part of the meaning that is planned for teaching (for example, what is proposed, concerning the particular object in the curricular guidelines), c) *implemented meaning* (what was finally taught to the students), and d) *assessed meaning* (content of assessment). In our research we try to identify the assessed meaning of probability in the entrance to university tests and compare this meaning with the intended institutional meaning for these students, as defined in the curricular guidelines we commented previously. This assessed meaning can be described by the mathematical objects linked to the problems proposed in these tests and the practices that students should carry out in their solution. The method of semiotic analysis proposed in the ontosemiotic approach is directed to identify these objects and the interpretive processes needed by the person who solves the problem. When this analysis is applied to the problems included in a test it serves to identify the correspondence between assessment (assessed meaning) and curricular guidelines (intended meaning), as well as to show the complex work required from the student in the test.

Previous Research

We also base on two previous research studies that have focused on the probability problems in textbooks, and served to identify the variables used in our investigation. Ortiz (1999) studied 22 high school mathematics textbooks and identified the main concepts implicit in the problems: compound experiments, dependence and independence, joint probability, total probability and the Bayes' theorem. In our analysis we will consider all these objects. Díaz (2004) studied the probabilistic problems in 17 university textbooks and classified the problems according to the following variables: a) type of experiment: sampling with or without replacement, or simple experiment; b) dependent or independent experiments; c) property or theorem used in the solution: product rule, computation of conditional probability from simple and compound probabilities, total probability or Bayes' rule; d) type of situations: synchronic (when experiments are simultaneous) or diachronic (consecutive experiments).

Although these two authors identified variables and categories that are relevant in the analysis of probability problems, they limited themselves to identify the presence or absence of each category in the sample of textbooks they analysed. Their studies did not highlight the complexity of the problems, since they did not perform a semiotic analysis of the same. The only previous analysis of the problems proposed in the entrance to university test is our previous paper (Contreras, López-Martín, Arteaga, & Carretero, 2015) where we just analysed the problems proposed in three different years and compared the presence of the mathematical objects identified by Díaz in her analysis in these years. Consequently our paper is a new contribution to this research question and complements these previous studies.

METHOD

The university entrance tests in Spain are currently regulated by the Royal Decree 1892/2008, of 14 November, where the conditions for access to official university studies and the admission procedures to Spanish public universities are described (MP, 2008). This document requires the students to pass a maturity test (university entrance test), in which the students' knowledge and abilities are assessed.

The mathematics test for Humanities and Social Sciences students consists of two options: A and B. The student can chose one of these options and should solve all the problems proposed in the selected option (mixing problems from both options is not allowed). The content of tests (4 problems) is similar for both options: the first problem is related to algebra, the second to analysis, the third to probability and the last to statistical inference. The correct solution to each problem is scored with 2.5 points from a total of 10 points in the full test. The content of the problems should be in agreement with the curricular guideline contents for high schools in Spain.

In our study, we focus only on the probability content of this curriculum. As we mentioned above, each test includes a specific problem related to this content. Moreover, as we will show in our analysis this problem is centred specifically on "Reinforcing ideas about prior and posterior probabilities, joint probability, conditional probability; total probability and the Bayes' rule".

In this study we examine the probabilistic problems contained in the tests proposed in the period 2003 to 2014. In each of these years, 6 different versions of the tests were used (depending on the semester and city); since each test has two options, we analysed a sample of 144 problems. Given that we only considered a Spanish region our study is exploratory, which is usual in qualitative research. We therefore do not intend to extrapolate the results to other different tests. However, we believe that our findings can serve to formulate some preliminary hypotheses about the probability content of the tests in other regions that would be tested in the final study.

RESULTS AND DISCUSSION

When analysing the mathematical objects needed to solve these problems we noticed that the problems proposed in these tests are quite complex and involve difficulty for students; below we summarise the main results concerning each variable analysed. Some problems in the test are used as examples and are included in the Appendix.

Type of Probability to be Computed

Most problems required the computation of more than a type of probability in different questions posed to the students, e.g. in the Example 1, students have to compute a compound probability in part a) and a conditional probability in part b). Globally 78% of the problems involved the computation of a conditional probability, which is difficult for students according to previous research (e.g. Pollatsek, Well, Konold, & Hardiman, 1987); 66% of the problems requested to compute a simple probability and 58% computing a compound probability. Furthermore, in some particular cases the students were asked to provide some proof for abstract properties of probability; e.g., in Example 2, the student need to justify, using formal proof if two events are independent or not. When studying the distribution of this variable over time (Figure 1), conditional probability is requested in at least half the problems proposed each year (6 or more problems), some years, like in

2011, 2012 and 2013 in almost all the problems.

Context of the Problem

We classified the problem context according the type of situations considered in the PISA assessment (OECD, 2010): a) Problems classified in the personal context category focus on activities of one's self, one's family or one's peer group, such as games, personal health, shopping or transportation (18%); b) Social contexts (38% of problems) focus on students' local or national community (e.g. Example 1 focus in a local geriatric residence); other context in this category include voting, public transport, demography or publicity; c) scientific problems, such as example 3 (3%) are related to the application of probability to science and technology; for example, climate, ecology or medicine; d) finally professional contexts (21%), such as the one used in Example 4, are centred on the world of work and involve questions related to production, quality control, accounting, or design, for example.

In addition to these contexts we also found a large proportion of problems (20%) where no application context is included (e.g., Example 2), which is contrary to current recommendations for statistics education and to curricular recommendations in Spain. These problems ask the student to transform or simplify algebraic expressions or provide justification of abstract probabilities rules. Probability knowledge and reasoning is needed in everyday or professional situations and the assessment instruments reflect the type of content we want the students to learn. In problems with no context, the multiple applications of probability to different sciences are hidden to the students and their probability reasoning is reduced to algebraic reasoning.

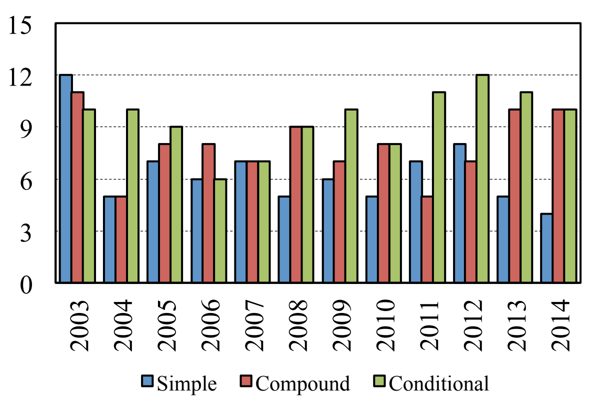


Figure 1. Type of probability requested

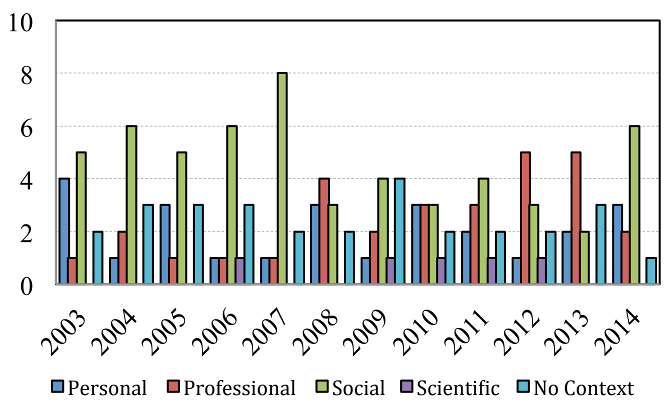


Figure 2. Context of the problem

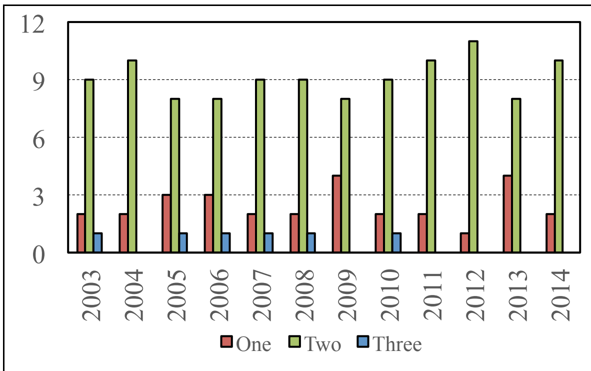
When considering the distribution of this variable over time (Figure 2) we find that social contexts are frequent in many years (as was also globally), which is natural, since the students that complete the exams are social science students; on the contrary scientific contexts are seldom found, what hide the applications of probability to science and technology.

Type of Experiment

Like in Ortiz (1999), problems include simple (20%) and compound experiments (80%). In this last case most problems in our study required the students to work with different simple experiments that should be combined (4% of problems involve three different experiments and 76% of them two different experiments). We can see this tendency in some examples in the Appendix: In Example 3, the scientists in the problem are women and men in different proportion (first experiment) and they vary also as regards their use of Internet (second experiment). In this problem as in many others, the experiments are subtly hidden to the student who should model the situation as a compound experiment. Moreover, as in this example 75% of compound experiments were formed from dependent experiments, which are more complex for the students according to Díaz (2004); only 25% of compound experiments were formed of independent experiments. The distribution of this variable over time is homogenous with predominance of two experiments per problem all the years (Figure 3).

Sample Space

The problem difficulty changes with the number and type of elementary events in the sample space, since in case of non-equiprobable elementary events the student should first find the probability for each elementary event before solving the questions posed in the problem. Only two problems in the sample considered a sample space with only two equiprobable experiments; 16% of the problems included more than two equiprobable events and 10% two non equiprobable events; 54% of sample spaces had more than two non equiprobable events and 19% of them consisted of abstract events (no probability given). This type of sample space is the most frequent in all the years analysed (Figure 4). Sample spaces with only two equiprobable events only appeared in 2004, while imprecise sample spaces are found almost all the years. Consequently, as regards the sample space, the problem difficulty is moderate or high (Ortiz, 1999) and the designers do not take into account the intuitive belief of many students in the equiprobability of results in a random experiment (Batanero, Serrano, & Garfield, 1996). This belief may result in assigning incorrect initial probabilities to the elementary event with the consequences of wrong solutions to the problem.



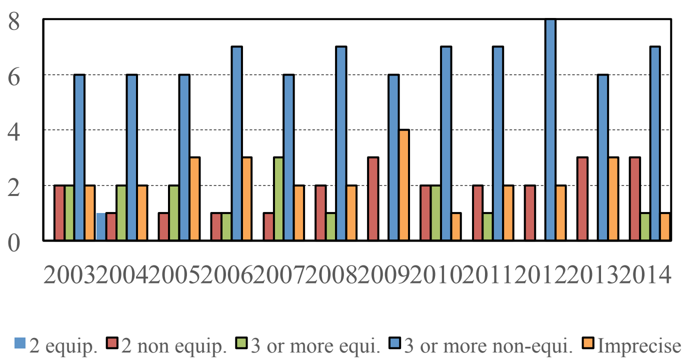


Figure 3. Number of experiments	Figure 4. Number and type of events in the sample space

Properties or Theorems that should be Applied

When computing a simple probability usually only the Laplace's rule is needed; however, when the probability is compound or conditional, and sometimes also in simple probabilities, the students have to remember and apply different probability rules to obtain the solution to the problem. We solved the problems and analysed which properties or theorems are needed to find the final solution to each of them. The result showed that all the problems proposed in the tests required that the student applied at least one decomposition of probability; usually the formula of conditional probability should be deduced from compound and simple probability (like in Example 2, part a); 69% of problems). The total probability (Example 3, part b); and the Bayes' theorem (Example 1, part b) which has proved to be difficult for students (e.g., Díaz, & de la Fuente, 2007) appeared in about 43% of the problems each; the union of non-exclusive events Example 3, part b) was needed in 40% of the problems. In Figure 5 we analyse this variable along time. Contrary to our expectations, the most frequently used theorem is the decomposition of conditional probability with no much variation over time.

Format of Data

Even when Gigerenzer (1994) remarked that the difficulties of problems diminish considerably when the data are given in absolute frequencies very few problems used this format. We found different types of data formats in the problem statement: absolute (natural) frequencies (Example 1; 33%); probabilities (Example 2; 25%), percentages (Example 4; 43%) and a mixture of several of these formats in the remaining problems. The type of format is very variable over time with more problems with data given in percentages in the last years.

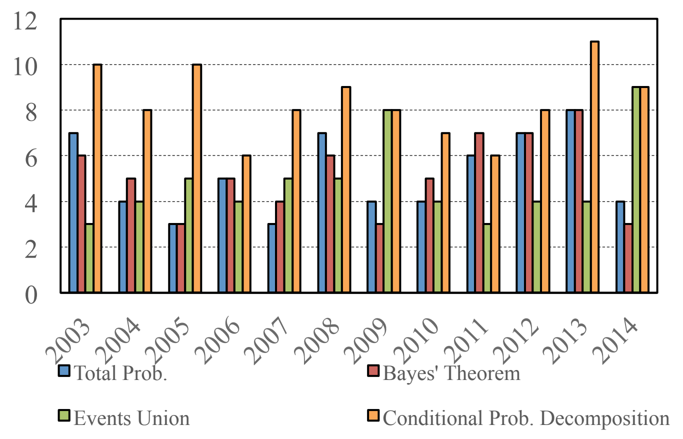


Figure 5. Theorems to be applied

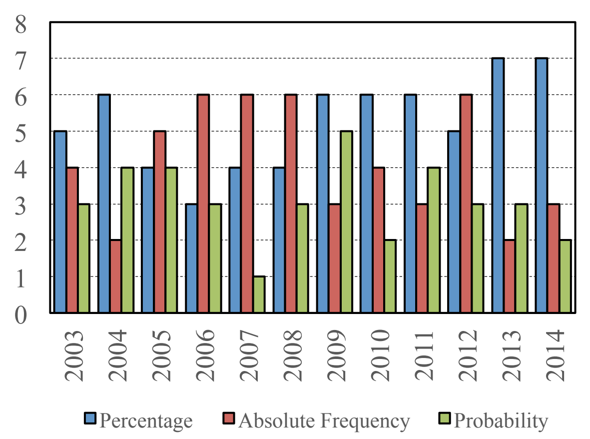


Figure 6. Data format

FINAL REMARKS

All the university entrance tests analysed include a probability problem (one of the four problems proposed to the students) related to compound or joint probability, and, therefore, these concepts are given great relevance in these tests. Since the score in the problem is 25% of the total score in the test, the correct solution to the problem affect the student's possibilities to choose their preferred career, as the universities select the students in order of punctuation in these tests.

Conditional probability problems are difficult, due to the large number of possibilities in which they can be presented. In several papers Huerta (e.g. Huerta, 2014) have classified in a theoretical what he calls "tertiary conditional probability problems". These problems provide three different data, involve at least one conditional probability either as data or as an unknown and all data and unknown in the problem are linked by ternary relationships, such as union, intersection or multiplication. The author analyse the variety of difficulty levels in these problems and suggest that students often confuse the type of problem, what may happen in the problems proposed at the University entrance tests and point to the high difficulties of these problems. The high school curriculum in Spain includes other probability content, such as the central limit theorem; the normal approximation to the binomial distribution; the law of large numbers. It also includes topics of arithmetic and algebra (one third of content), as well as of calculus (another third of content). The tests also include another problem related to inference (statistical tests or confidence intervals); it is clear stochastic reasoning (probability and inference that suppose half the score in the test) is given high relevance in the tests, and in particular joint and conditional probability, which is the most frequent content in the probability problem.

Other results point to the high difficulty of the probability problems proposed in the university entrance tests: For example, many problems present data using probabilities of percentages or mix different format, what influence the problem difficulty and the way the students approach to the problem (arithmetic or probabilistic) according to Huerta and Lonjedo (2006). Most problems are based on compound sample spaces with non-equiprobable events and dependent experiments.

Since the scores obtained by the students in the University entrance tests often determine that they can enter their preferred career; our results should be taken into account by the test designers in order to build more reasonable assessment tests in the future.

Acknowledgement: Proyecto EDU2013-41141-P (MEC), y Grupo FQM126 (Junta de Andalucía).

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APPENDIX: EXAMPLES OF PROBLEMS PROPOSED IN THE TESTS

Example 1 (2003, test 5, option A). From a total of 212 elderly people in a geriatric home, 44 of them have lung problems; 78 of them are regular smokers, and only 8 have lung disease and are non-smokers.

a. What is the probability that an elderly person in this home, chosen at random, does not smoke and does not have lung disease?

b. What percentage of elderly people with lung problems are smokers?

Example 2 (2003, test 6, option A). The probabilities for the following events linked to the same random experiment are given: $P(B) = 0.7$, $P(A|B) = 0.8$, $P(A \cap \bar{B}) = 0.24$.

c. Compute $P(A \cap B)$; b. Compute $P(A)$. c. Determine whether A and B are independent.

Example 3. (2012, test 1, option A). In a scientific conference 200 participants are asked if they buy their travel tickets on Internet. 120 participants are men and 84 of them buy their travel tickents on Internet, while 24 women do not use Internet to buy their tickets. We select at random a participant in this conference. Find the following probabilities:

a. The person does not use Internet to buy their tickets.

b. If the person is a woman, she uses Internet to buy her tickets

c. That the person is a man, when we know the person has used Internet to buy his or her tickets.

Example 4.(2013, test 4, option A). In a health center, two therapies for stopping smoking A and B are compared. From all the people that attend the center to stop smoking, 45% of them follows therapy A and the remaining patients follow therapy B. After a year was spent with the therapies, 70% of patients following therapu A and 80% of those following therapy B stop smoking. We select a patient of the center at random. Compute the following probabilities:

a. The person stopped smoking after one year.

b. If the person stopped smoking after one year, the person followed therapy A.

c. If the person continue smoking afteer one year, the person followed therapy A.