Hands-on Modelling with Wason Cards and Tinker Cubes: First steps in logical and Bayesian reasoning in fourth grade

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Historical Background

Thesis (Laplace): Probabilistic reasoning is not just the normative framework for rational decisions but also “common sense reduced to a calculus”

Antithesis (e.g., Tversky & Kahneman): The human mind is not “hardwired“ for probabilistic reasoning

Synthesis: The modern view in cognitive psychology is that humans‘ capability to reason probabilistically often depends on the representation of information (e.g., Gigerenzer)
Some examples …

1) Bayesian Reasoning (HIV-Test)
2) Conjunction fallacy (Linda Task)

All examples are known as famous cognitive illusions.
However, „natural“ frequencies can help in each case …

3) Can the natural frequency approach even be applied with 4th graders?
1) Natural Frequencies

10,000 clients

1 HIV

1 positive

0 negative

9,999 no HIV

1 positive

9,998 negative

Conditional Probabilities

\[ p(\text{HIV}) = 0.01\% \]

\[ p(\text{positive} | \text{HIV}) = 99.9\% \]

\[ p(\text{negative} | \text{no HIV}) = 99.99\% \]

\[ p(\text{HIV} | \text{positive}) = \frac{1}{1 + 1} \]

\[ = \frac{0.0001 \times 0.999}{0.0001 \times 0.999 + 0.9999 \times 0.001} \]
2) Conjunction Fallacy

Linda is 31 years old. She majored in philosophy and was active in the anti-nuclear movement. When Linda was in highschool she was outspoken.

Which statement is more likely to be true:

a) Linda is a bank teller
b) Linda is a bank teller and a feminist
Making cognitive biases disappear with frequencies…

Linda is 31 years old. She majored in philosophy and was active in the anti-nuclear movement. When Linda was in highschool she was outspoken. Imagine 100 women that satisfy this description.

How many of them are

a) Bank tellers
b) Bank tellers and feminists
3) Can 4th graders be introduced to the basic ideas of Bayesian Reasoning?
Hands-on analogue modeling with Tinker Cubes
The boys and the girls
Lively urns and eventful trees
Kino
120

76  44
52  24 17  27

Fußball
References:

Empirical results: Kurz 2008, Martignon, (JRME, in revision)

Martignon & Kurz-Milcke, 2007

Martignon & Krauss, 2008

Kurz-Milcke, Gigerenzer & Martignon, NYAS 2008