ICME-TSG 13
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Concrete to Abstract in a Grade 5/6 Class

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Ingredients in the Research

- Interest in ICT in the classroom to enhance learning
- Graham Jones’ call for research on students’ connections between classical and frequentist probability
- Availability of *TinkerPlots* Sampler
- A grade 5/6 with interested teacher willing to work with student researcher
The Mixer and the Spinner in *TinkerPlots* Sampler
Concrete to Abstract conceptual framework

Concrete

Experimental probability

Conceptually concrete/abstract

The Connection

Law of large numbers

Abstract

Manipulatives

Simulator

Theoretical probability
Lesson 1 with 27 Grade 5/6 Students

• Structured discussion of probability
  – events
  – chance phrases
  – “random”, “variation”

• Coin tossing
  – 10 trials
  – theoretical probability (“half”)
  – experimental probability
  – students tossing 10 times, recording
  – combining results
Tracking the tossing of a coin by students
Lesson 2 with 27 Grade 5/6 Students

- *TinkerPlots* Sampler to consider connection with larger trials
- Trials of 100, 1000, and 10,000
- Mixer, running tally, graph of variation from expected, number of heads in a row
Tracking the simulated tossing of a coin
Screen dump of the Sampler whole class coin activity including three graphs
Student written reflections on …

- What is experimental probability?
- What is theoretical probability?
- How are they connected?
- Responses to these questions helped to confirm the choice of students for interview.
Student Interviews [5 In-depth]

- *TinkerPlots* Sampler for a die modeled with the spinner.
- Students asked to
  - explain
  - generalise
  - find evidence
  - apply current understanding
  - draw graphs for 10, 20, 100, 1000 die tosses
- Three dice scenarios (spinner set up hidden)
  - no loading
  - large loading
  - slight loading
Screen dump of the Sampler personal interview dice activity including a table and graph.
Student Interviews

• Interview ended with a discussion of the relationship between experimental and theoretical probability as experienced during the interview.

• The level of student response was based on mappings of the ideas expected to be included in the students’ responses in relation to the five elements of the framework.

• The way the components of the figures were combined was assessed using the SOLO model of Biggs and Collis.
Levels of student understanding

• Unistructural responses (U): single aspects of the element and a lack of recognition of contradictions.

• Multistructural responses (M): a series of aspects of the element with contradictions likely to be recognized but unresolved.

• Relational responses (R): a linking together of aspects of the element, resolving to a large extent conflict that arose.
Mapping in relation to each of the five elements in Concrete to Abstract Framework
## SOLO Levels for the Five Elements for Five Interviewed Students

<table>
<thead>
<tr>
<th></th>
<th>Manipulatives</th>
<th>Simulator</th>
<th>Experimental prob.</th>
<th>Theoretical prob.</th>
<th>Law of large No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>R</td>
<td>R</td>
<td>M</td>
<td>M</td>
<td>R</td>
</tr>
<tr>
<td>S2</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>M</td>
<td>U</td>
</tr>
<tr>
<td>S3</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>S4</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>M</td>
<td>U</td>
</tr>
<tr>
<td>S5</td>
<td>R</td>
<td>R</td>
<td>M</td>
<td>M</td>
<td>U</td>
</tr>
</tbody>
</table>
Connections among five elements in Concrete to Abstract Framework

Concrete

- Experimental probability
  - A probability estimate, of an event, can be calculated through the observed outcomes of an experiment
- Manipulatives
  - Symbolically represents an object

Conceptually concrete/abstract

- As the sample increases, a pattern forms
- Simulates the internal processes
- Symbolically represents an object

Abstract

- Law of large numbers
  - Facilitates the rapid collection of a large number of trials, of a simulated event
- Theoretical probability
  - Simulations are based on theoretical underpinnings.

As the number of trials approaches infinity, the results should reflect the theoretical probability
**SOLO Levels for the connections made among the 5 elements**

<table>
<thead>
<tr>
<th>Student</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>R</td>
<td>U</td>
<td>R</td>
<td>M</td>
<td>U</td>
</tr>
</tbody>
</table>
Summaries of students’ observed understanding

• S1: discussed smoothing out of results; larger trials determined fairness (R)
• S3: as for S1, plus used decimals to track experimental proportions approaching theoretical probability (R)
Summaries of students’ observed understanding

• S2/S5: discussed levelling out for large trials but short term variation in experimental outcomes shows fairness (U)

• S4: suggested probability (in graphs) based on decimal (.167) for all sample sizes; surprised by variation in small sample simulations (M− could not resolve conflict)
Discussion and Further questions

• Importance of proportional reasoning.
• Link of fairness and equally likely outcomes.
• Scaling of graphs plays a large roll in conceiving the leveling out of data with large samples.
• *TinkerPlots* Sampler a meaningful link – is it truly random?
• Need explicitly to map the connections for students.