METACOGNITION IN LEARNING
ELEMENTARY PROBABILITY AND STATISTICS

A dissertation submitted to the

Division of Research and Advanced Studies

of the University of Cincinnati

in partial fulfillment of the

requirements for the degree of

DOCTOR OF EDUCATION (Ed.D.)

in the Department of Curriculum and Instruction

of the College of Education

2004

by

Teri Rysz

B.S., University of Cincinnati, 1979

M.Ed., University of Cincinnati, 1999

Committee Co-Chairs: Dr. Janet C. Bobango

Dr. Daniel D. Wheeler

UMI Number: 3159784

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

Abstract

This study used qualitative research methods to identify metacognitive thoughts adult students had while learning elementary probability and statistics concepts and while problem solving, alone and with other students. From the 49 students observed in a classroom setting, seven were purposefully selected to be interviewed outside the classroom three times: a review of the student's notes taken during a class immediately preceding the interview, the student solving a problem alone, and a group of three or four students solving a problem together.

Classroom observation notes were organized according to categories of metacognitive thinking—orientation, organization, execution, and verification—and a fifth category labeled "lack of metacognition." Interviews were recorded, transcribed, and coded according to the same categories. During data analysis four themes found in the literature emerged from the data: novice vs. expert problem solving, statistics as a viable subject, self-reporting, and a cognitive-metacognitive framework.

The interviewed students could be classified into two groups by similar characteristics regarding the themes. It was found that students can earn above-average grades using limited or no metacognition, but those who provided evidence of cognitive awareness and self-monitoring were better able to report an understanding of probability and statistics concepts.

Acknowledgements

Thank you Dr. Bobango for guiding me to my career as a mathematics educator and through the abundance of hours I have taken to complete my education this far.

Thank you Dr. Wheeler for the support and confidence you gave me through my coursework and writing process.

Thank you Dr. Markle for your academic support and encouragement to finish.

Thank you Dr. Emenaker for sacrificing your time to my goals.

Thank you to all the people who provided the opportunity for me to collect data for this study.

Thank you Mike for your super-natural patience and support while I nurtured my soul.

Thank you Jonathan and Henry for giving up our time so that I could pursue my goal.

And, Steve, finally I "just did it."

But by the grace of God I am what I am, and God's grace toward me has not been in vain.

—1 Corinthians 15

Table of Contents

List of Tables

Chapter One

Problem Definition

Adult students who enroll in an elementary probability and statistics course bring identified characteristics to the classroom. Many have weak mathematical and/or reading skills (Ainley & Pratt, 2001) which confound into anxious feelings toward mathematics classes. Often, students taking the class believe elementary probability and statistics is a collection of theories that are not relevant to their lives, academic or otherwise (Beitz & Wolf, 1997). Passing the course is just another process to meet program requirements. Students' common misconceptions, developed through life experiences, have been reported in several studies (Batanero, Godino, Vallecillos, Green & Homes, 1994; Fischbein, 1975; Kahneman and Tversky, 1982; Shaughnessy, 1992; Tversky & Kahneman, 1971; Well, Pollatsek, and Boyce, 1990). These studies identify students' intuitions that "have been distorted and influenced by experiences other than instruction" (Shaughnessy, 1992, p. 486). It is not known when or how the misconceptions develop, which makes changing them a difficult task.

Until the 1980s, probability and statistics had not been considered an important branch of mathematics for all students to study; calls to reform mathematics education in primary and secondary schools drew attention to the usefulness of stochastic information for all ages (National Council of Teachers of Mathematics, 1980, 1989; National Research Council, 1989). The Mathematical Association of America (MAA, 1998) and the American Mathematical Association of Two-Year Colleges (Cohen, 1995) supported the increased importance of probability and statistics at the post-secondary level. Efforts to improve instruction in probability and statistics concepts have not increased success students experience in learning theories presented in the course (Shaughnessy, 1992). To better understand why students make similar mistakes while learning to make inferences about populations based on collected data, it is necessary to look beyond cognitive factors, including metacognition (Lester, 1989).

What role does metacognition play when students are learning how to make decisions under conditions of uncertainty? Students' explanations about probability and statistics concepts indicate a need for further research to address common mistakes made, despite efforts to improve instruction. Elementary probability and statistics curricula focus on data analysis and probability theory to build inferential skills, specifically extrapolation and prediction. Many theories and formulas are presented as tools for answering questions regarding situations about which students have not yet been exposed. Answering novel, quantitative questions categorizes the subject as mathematical problem solving. Metacognition is a skill often associated and studied with problem solving which in itself has been difficult for many students for various reasons; learning to make decisions about collected data introduces additional problem solving difficulties. Because probability and statistics is a problem solving arena, this is an appropriate factor to explore. Shaughnessy (1985) reported that probability and statistics problems are often absent from problem solving research and believes this situation should change because "metacognitive aspects are brought into sharp focus in problems involving probability and statistics . . . [and] . . . stochastic problems are good examples of applied problems" (p. 409). Is metacognition a necessary element for success in undergraduate elementary probability and statistics courses? If the students are aware of their own cognitive strengths and weaknesses and use them to monitor progress during problem solving, will they better understand probability and statistics concepts taught in undergraduate courses?

Background

Long before the word “metacognition” was coined sometime in the 1970s, people interested in how humans think, how the mind works, and how students learn wrote about the importance of cognitive thoughts. Descartes (1952), Spinoza (Weinert, 1987), and Locke (Brown, 1987) were three 16th and 17th century people interested in what others knew about their own knowledge, what they thought about thinking. For many years other perspectives on learning, such as behaviorism, pushed cognitive factors to the background (Schoenfeld, 1992). In the late 1800s and early 1900s some researchers, including Wundt and Thorndike, studied empirical cognitive behavior. As scientists began to develop artificial intelligence concepts during the 20th century, interest in how people accomplished specific tasks led to empirical studies in how the mind works (Minsky, 1968). Tulving and Madigan initiated the research field with meta-cognitive processes in their investigations into human memory (Campione, Brown, & Connell, 1989), and John H. Flavell (Flavell, Friedrichs, & Hoyt, 1970) transferred the interest in what humans know about their own memory to what humans know about their own cognitive processes. Today’s continued interest in improving mathematics education demands research in cognitive activities important to problem solving.

Pólya’s (1957) widely accepted phases of problem solving—understanding the problem, planning a method of solution, carrying out the plan, and verifying the answer derived—implied thinking about cognitive skills. Schoenfeld (1983) developed six transitional points at which thinking about cognition could be identified by the problem solver—reading the problem, analyzing what needs to be done, exploring different possibilities, planning the best solution, implementing the plan, and verifying the answer is an appropriate solution. Schoenfeld (1987) reported that explicit instruction on how to monitor progress was employed to teach problem solving skills to college students. His questions, "What (exactly) are you doing? (Can you describe it precisely?), Why are you doing it? (How does it fit into the solution?), [and] How does it help you? (What will you do with the outcome when you obtain it?)" (p. 206), improved problem solving success. His work with students solving calculus, algebra, and geometry problems in small groups revealed the amount of thinking invested in appropriate and inappropriate solutions. In order for the subjects to develop into expert problem solvers (those who self-regulate their progress), they needed explicit instruction on how to develop a critical viewpoint (Schoenfeld, 1992). Lester, Garofalo, and Kroll (1989) developed a cognitive-metacognitive framework for determining appropriate mathematical problem solving instruction with seventh-grade students. They found that facilitating students' metacognitive development through teacher modeling contributed to success during problem solving.

Shaughnessy (1985) found that teaching successful problem solving requires the students' awareness of their thinking about cognition and suggested that self-monitoring and regulation should become a priority for instructors. "Metacognitive aspects are brought into sharp focus in problems involving probability and statistics" (p. 409); courses become problem solving in a statistical domain which requires previously learned mathematical knowledge. Meaning is constructed through reflection, and metacognition is specific thought about one's own cognitive processes and how well progress is being made in solving problems. "For these reasons we believe there is much to be gained from studying the implications of probabilistic problem solving for general problem solving" (p. 410).

A fundamental tenet of the constructivist theory of learning is people learn in different ways (von Glassersfeld, 1996), using different strategies and goals; some learners may be using metacognitive thoughts while others are not. When asked to self-report mental processes, subjects may have difficulties either in recognizing their own thoughts or in communicating them to others. Subjects may say things they think they are expected to believe about the situation at hand but do not extend to other life situations (Ericsson & Simon, 1993).

What is considered to be important in mathematics education has been evolving throughout history. Probability and statistics concepts are now included in mathematical study from kindergarten through high school as well as in college and are slowly being accepted as important mathematics for every student to learn (Shaughnessy, 1992). Because this field of mathematics is a problem solving arena, it lends itself to the study of how students work through novel situations. What do students report about their own thinking while learning to make appropriate inferences about populations?

Importance of the Research

Knowledge in any domain is developed to be used, not just known (Greer, 1996). People gain knowledge to become members of a community and to be competent in a domain; they are working toward becoming experts (Hatano, 1996). If educators pose appropriate situations and then encourage reflection and interactive mathematical communication, students can determine for themselves the most effective paths for becoming adaptive experts. Students who learn to solve problems independently have evolved from novice learners to expert users of knowledge.

Some students enter college with deficiencies in mathematics and reading comprehension, which often results in math anxiety and little skill in knowing how to build on prior knowledge. Perhaps they need more guidance in how to actively use previously acquired mathematical knowledge to better learn—to understand—concepts for elementary probability and statistics. "If the critical role of metacognition can be made more clear, educators will be able to incorporate metacognitive aspects into mathematics instruction" (Garofalo & Lester, 1985, p. 172). If it can be shown that self-monitoring of progress during problem solving results in students successfully learning probability and statistics concepts, mathematics educators could be better informed about the importance of identifying students' metacognitive thoughts.

Problem Statement

If students are aware of their own cognitive strengths and weaknesses and monitor their progress during problem solving, will they be more successful in learning elementary probability and statistics concepts taught in undergraduate courses? If using metacognition is important for understanding probability and statistics, educators could be better informed about classroom techniques for instructing their students. The objective of this qualitative research study was to investigate students' use of thought processes to successfully solve statistical problems. The following questions were addressed in designing, implementing, and analyzing this study.

1. What are the students doing cognitively to learn concepts in elementary probability and statistics?
2. What role does metacognition play when students are learning how to make decisions that require an understanding of probability and statistics concepts?
3. Is metacognition a necessary element for success in undergraduate elementary probability and statistics courses?

Before this study commenced, it was believed that college students who provide evidence that they are aware of their own learning strengths and use them to actively monitor progress in probability and statistics problem solving are more successful in learning the concepts than the students who could be considered non-experts.

Definition of Terms

Explanations of terms pertinent to this study follow.

1. Cognitive-metacognitive framework: a specific list of categories of types of metacognitive thoughts a student may have during problem solving. The categories are orientation, organization, execution, and verification (Garofalo & Lester, 1985).
2. Expert vs. novice problem solver: two categories of problem solvers. An expert pays attention to the structure of a problem and poses critical questions while working through novel situations. A novice

relies on intuition and feelings, rarely monitoring progress toward an appropriate solution (Schoenfeld, 1987).

3. Metacognition: a person's knowledge concerning cognitive processes and results that might effect learning.
4. Elementary probability and statistics: a study of collecting and representing numerical facts, including relative frequencies, and the interpretation of data as a tool for prediction or inference.
5. Problem solving: the process of using various types of knowledge to answer a question to which the answer is not readily available.
6. Stochastics: a word used to refer to the combined study of probability and statistics.
7. Successful student: someone who earns a C or better on individual assessments or as a course grade.

Overview of the Study

This study was conducted in one section of a college level elementary probability and statistics course at a midwestern university during the ten-week, spring quarter of 2003. All the sections of this course were large (initial enrollment of 56 students in the observed section) and primarily lecture in structure. Qualitative research methods were employed to make visible the covert thought processes probability and statistics students had while learning and solving individual and group problems. Students were observed in their class setting, and seven of the students were purposefully selected to explain the notes they took during one of the classes. Then the selected students were asked to solve, out loud, two statistical problems, one alone as the researcher observed and one with a group of the participating students. All of the students could not be interviewed, which eliminates generalizing the results to all statistics students. The process of selecting the interviewees, however, did provide a sample representative of the students in this course section. The interviews were taped, transcribed, coded for evidence of metacognitive behavior in these seven human subjects, and analyzed. Four common themes emerged from the individual profiles of the interviewed participants: novice vs. expert problem solving, statistics as a viable subject, self-reporting, and the cognitive-metacognitive framework.

Overview of the Chapters

The following chapters describe in detail why and how this study proceeded and conclusions that resulted from analyzing the data. Chapter two is a literature review of metacognition: studies that recognized the importance of metacognitive thoughts, especially in problem solving instruction; statistics students and their intuitions; and common misconceptions brought to the statistics classroom. Grounding metacognition and its use in the constructivist theory of learning points the way to understanding how metacognition might or might not help students understand theory presented in the college elementary probability and statistics classroom. Chapter three describes the methodology used in this study. The participants, the course, and the interviews in which the students participated are described in detail. Chapter four presents the results of the classroom observations and the three types of student interviews. Notes taken during the classroom observations are presented by the cognitive-metacognitive framework categories. Student profiles are then presented in order of the two problem solving groups and by the first midterm exam scores. Each of the profiles is organized by the themes that emerged from analyzing the qualitative data. Chapter five summarizes the study with respect to the three guiding questions addressed in this study (What are the students doing cognitively to learn concepts in elementary probability and statistics? What role does metacognition play when students are learning how to make decisions that require an understanding of probability and statistics concepts? and Is metacognition a necessary element for success in undergraduate elementary probability and statistics courses?) and discusses possible further research gleaned from the results.

Chapter Two

Literature Review

In order to be better teachers, educators examine how students learn, utilize pedagogy to address the students' needs, and carry out methods of instruction that encourage construction of knowledge into a viable organization of facts and procedures. Grounding planning and instruction in the constructivist theory of learning, may contribute significantly to excellent education. This chapter discusses the importance of constructivism as it relates to metacognitive processes that may be an important tool for college students learning introductory probability and statistics concepts. History of the development of the study of metacognition is described, leading to current thoughts on how students can be guided into developing a mathematical viewpoint necessary for understanding elementary probability and statistics concepts. Because undergraduate college students are expected to develop both an overall comprehension of probability and statistics concepts and a more itemized awareness of understanding pieces of information and strategies, pedagogy that supports and develops metacognitive skills is examined, leading to a viable framework for identifying students' covert cognitive thoughts.

Constructivism

Although all constructivists do not agree on the primary focus of learning, the principal tenet of "constructivism generally emphasizes that students construct knowledge by themselves, not by swallowing ready-made knowledge from the outside" (Hatano, 1996, p. 211). Facts can be passively communicated but meaning cannot. In general, constructivists do agree that 1) knowledge is acquired by construction, not just by transmission; 2) knowledge organization is restructured when new knowledge is acquired; 3) knowledge is constrained internally and externally; 4) knowledge is acquired by domain; and 5) knowledge acquisition is situated in contexts (Hatano, 1996). Through social experiences and reflection on the experiences, learners reorganize their schemas of information to assimilate new concepts into viable knowledge. Whether or not true knowledge can be attained and the importance of social interaction in the construction of knowledge are the two pieces that are not always agreed upon among constructivists. Ernest (1996) outlines the differences in four constructivist paradigms: information-processing, weak constructivism, radical constructivism, and social constructivism. Information-processing and weak constructivism state that there is an absolute and attainable truth where radical and social constructivism follow a fallibist epistemology. He summarizes each paradigm with metaphors for the mind and world models shown in Table 1 (p. 344).

Ernest (1996) continues descriptions of the four paradigms by defining the most important aspect of learning in each view, which could be different for each probability and statistic student. The information-processing constructivist focuses on how humans process information, including the acquisition, storage, and retrieval of knowledge (p. 338). Students connect new information to previously constructed knowledge to assimilate the new knowledge and/or to solve problems by using the previously constructed knowledge. Weak constructivism promotes the idea that learners construct truths that can only be known by means of their world experiences (p. 339). Through participating in class and completing homework, students broaden experiences with

Table 1.

Constructivist metaphors for mind and world models

Type of Constructivism	Metaphor for the Mind	Model of the World
Information-Processing	Computer, unfeeling thinking machine	Newtonian Absolute space with physical objects (Scientific Realism)
Weak	'Soft' computer (brain-as-machine)	Newtonian Absolute space with physical objects (Scientific Realism)
Radical	Evolving, adapting, isolated biological organism	Student's private domain of experience
Social	Persons in conversation	Socially constructed, shared world

Note. From *Theories of mathematical learning* (p. 344). by P. Ernest, 1996, Mahwah, NJ: Lawrence Erlbaum.

newly acquired statistical information. Radical constructivism places priority on the function of cognition; constructed knowledge serves to organize world experiences rather than to arrive at an absolute truth (p. 340). Students might begin to realize that probability and statistics could be useful for acquiring knowledge in other domains. Social constructivism centers on knowledge as an organization of socially accepted concepts (p. 342). Students organize their knowledge according to what the experts have already constructed. In summary, constructivism is concerned with learning, not teaching (Greer, 1996, p. 183); it requires critical reflection on what is socially accepted as truth.

All four paradigms are explored when investigating students' use of metacognition in constructing stochastic knowledge. Qualitative data collection procedures elicit explanations of how previously constructed knowledge is retrieved to assimilate the new information (information-processing). Importance is placed on learners' reflections on social experiences presented in the classroom (weak). Students who express the benefits of learning statistical concepts in domains outside the classroom are organizing their world experiences (radical). Students' reflections about the concepts currently accepted by the mathematics community serve to organize new concepts with old (social).

Well-developed metacognitive skills can be a tremendously helpful tool in all learning. Following constructivist principles, they may not be essential for all students. Everyone is different. The same event is interpreted differently by any two people because they bring different subjective experiences to the current situation. "Sharing meaning, ideas, and knowledge, therefore, is like sharing an apple pie or a bottle of wine: None of the participants can taste the share another is having" (von Glassersfeld, 1996, p. 311). No solution is absolute truth; there are many approaches to learning.

Metacognition

History

Descartes spent his life (1596-1650) searching for "truth." In his writing titled "Rules for the Direction of the Mind" (Descartes, 1952) he listed guidelines for this search. In one of the rules he declared that, "If in the matters to be examined we come to a step in series which our understanding is not sufficiently well able to have an intuitive cognition, we must stop short there . . ." (p. 12). In another rule he continues this thought with "If we don't understand something it helps to draw pictures or make a symbolic representation. This keeps easy facts clearly stated while we concentrate on more complex ideas" (p. 33). This set of rules was a very early predecessor to Pólya's heuristics for problem solving (Schoenfeld, 1992, p. 345). Descartes was one of the first philosophers to acknowledge the importance of examining one's own cognitive processes in order to reach a purpose or goal. He believed earlier philosophers, such as Papus, Plato, and Aristotle, purposely omitted communication of their own cognitive processes in order to keep us in awe of their greatness, and they were wrong for doing that (Descartes, 1952, p. 5).

In addition to Descartes, Spinoza, who lived from 1632 to 1677, contemplated thought processes and has been quoted as saying "Also, if somebody knows something, then he knows that he knows it and at the same time he knows that he knows that he knows" (Weinert, 1987, p. 1). John Locke (1632-1704) defined reflection as the "'perception of the state of our own minds' or 'the notice which the mind takes of its own operations'" (Brown, 1987, p. 70); the very young and the uneducated have not learned because they have not learned to reflect. Around 1880 Wilhelm Wundt, an empiricist psychologist, used experimentation and self-reporting to study thought processes scientifically (Schoenfeld, 1992).

Before the word metacognition was coined, developmentalists such as Dewey and Piaget acknowledged that children learn by doing and by thinking about what they are doing in their studies about mental processes (Kirkpatrick, 1985, p. 10). When Pólya (1957) developed his heuristics for problem solving he was outlining ways for students to reflect on their progress and to assess the successfulness of the procedures used. He was providing "metacognitive prompts" for awareness of knowledge about problem solving and monitoring of work completed (Lester, 1985). Vygotsky's theory of internalization and zone of proximal development, described in *Mind in Society*, is closely related to the regulation part of metacognition (Schoenfeld, 1987, 1992). In addition, according to Silver (1985), many researchers have been interested in metacognitive skills but labeled them as "control processes," "Test Operate Test Exit/TOTE," "reflective intelligence," and "executive scheme."

Another predecessor to metacognitive studies was Thorndike's (1917) study of 6th graders' errors in reading paragraphs. He reported that students read passages and failed to monitor their comprehension and even stated that they understood the reading whether they did or did not. He compared the novice students' mistakes in comprehension to the thoughts an expert reader might have while reading. The students would correct their mistakes if they were pointed out, but "they do not, however, of their own accord test their responses by thinking out their subtler or more remote implications" (p. 331). Thorndike's work on types of courses that improve the ability to think has had an impact on research in areas leading to mathematical cognition (Schoenfeld, 1992, p 346). He found that effect size of improved thinking was not due to types of courses studied (*i.e.*, mathematics and languages), the then traditional point of view, but that "Those who have the most to begin with gain the most during the [school] year" (Thorndike, 1924, p. 95). Good thinkers became better thinkers no matter what subject they studied.

Another area of research that began in the 1950s with the invention of computers—artificial intelligence—refuted importance of the then popular behaviorist movement and renewed study of cognition, focusing on metacognitive skills. Information processing looked at the structure of memory, knowledge representations and retrieval processes, and problem solving rules. In a preface to a collection of edited Ph.D. theses Minsky (1968) defined artificial intelligence as "the science of making machines do things that would require intelligence if done by men" (p. v). Minsky explained that in order to make non-cognitive computers process cognitive information, researchers had to go beyond the behaviorists' point of view—input-output observables—to mentalists' descriptions of thought processes, which could also be called human cognition skills. This new focus on the importance of human cognition supported the importance of humans reflecting on their cognitive processes (metacognition), but ". . . it was not until the early 1980s that control and other aspects of metacognition began to be a focus of attention for mathematical problem-solving researchers" (Lester, 1994, p. 671).

Tulving and Madigan initiated the research field with metacognitive processes in their investigations into human memory (Campione, Brown, & Connell, 1989) and John H. Flavell (Flavell, Friedrichs, & Hoyt, 1970) transferred the interest in what humans know about their own memory to what they know about their own cognitive processes. He is credited by many cognitive researchers (Brown, 1987; Campione, Brown, & Connell, 1989; Lester, 1985; Schoenfeld, 1992) as the "Father of Metacognition." His somewhat lengthy description of metacognition is often cited as a starting point for studies in mathematical problem solving (Garofalo & Lester, 1985; Lester, 1985; Schoenfeld, 1985, 1992).

> Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them, e.g., the learning-relevant properties of information or data. For example, I am engaging in metacognition (metamemory, metalearning, metattention, metalanguage, or whatever) if I notice that I am having more trouble learning *A* than *B*; if

> it strikes me that I should double-check *C* before accepting it as a fact; if it occurs to me that I had better scrutinize each and every alternative in any multiple-choice type task situation before deciding which is the best one; if I become aware that I am not sure what the experimenter really wants me to do; if I sense that I had better make a note of *D* because I may forget it; if I think to ask someone about *E* to see if I have it right. Such examples could be multiplied endlessly. In any kind of cognitive transaction with the human or nonhuman environment, a variety of information processing activities may go on. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective (Flavell, 1976, p. 232).

In addition, Flavell (1987) outlined three categories of metacognition with person variables (intra-individual, inter-individual, and global), task variables, and strategy variables. Person variables include information about what we know about ourselves and about others when learning. Task variables are knowledge about a specific domain's concepts. And strategy variables are what we know about manipulating domain concepts to answer a question. "Metacognitive knowledge involves the interaction of person, task, and strategy" (Garofalo & Lester, 1985, p. 168). According to Flavell (1987), metacognition is helpful for any organism that thinks a lot; makes mistakes, needing self-regulation to correct; wants to communicate with other organisms; needs to plan ahead; makes decisions; and/or needs to explain phenomena (p. 27). His reflections connect metacognitive problem solving skills to the constructivist learning theory. Both place importance on reflection and critical thinking within the social realm of learning.

Today the leaders in metacognitive research agree that metacognition is an area that needs more research in domain specific studies (Kluwe, 1987; Lester, 1989; Shaughnessy, 1985, 1992). Cognitivists recognize that the problems students have in developing mathematical proficiency, a primary concern of mathematics education advocates, cannot be answered through intellectual concepts alone (Garofalo & Lester, 1985). Because we have not been able to pinpoint the reasons only some students are successful in mathematics, we must go beyond strictly cognitive issues; other factors, that could include affective and metacognitive skills, may be involved as well. "Training in the use of a collection of skills and heuristics without attention to affective and metacognitive aspects of problem solving is inadequate" (Lester, 1989, p. 117).

Various methods for understanding the parts of cognition involve, among other things, the study of metacognition and affective beliefs. In an intervention program for students who had failed to learn the required mathematics for their biology program, Zan (2000) found that instruction in metacognitive skills improved attitude towards mathematics which in turn improved performance on assessments. Maqsud (1998) reported that attitude toward mathematics has a positive correlation with mathematics achievement and utilized individual discussions with students to emphasize the importance of monitoring progress in problem solving. His results showed that individualized, tailored teaching methods enhanced mathematics attitude and subsequently improved achievement with a group of students who previously did not score well on a mathematics achievement assessment. Anthony's (1996) study about active vs. passive learning concludes that students' academic goals affect their learning strategies and "to cope with the high level of cognitive, metacognitive, affective and resource management demands students must develop expertise in how to learn and use that expertise to construct useful knowledge" (p. 365). These studies recognize the importance of looking beyond purely cognitive aspects in the processes of coming to know mathematics.

Epistemology

Epistemology is the theory of knowledge, and "one's world views/set of beliefs/epistemology determines the kind of meta-reasoning one uses" (Schoenfeld, 1985, p. 368). The level of mathematical thinking that a student develops is dependent on his level of metacognitive skills. If a student is able to think mathematically he has developed a mathematical point of view which means he values the processes of describing intuitions mathematically and has the predilection to apply them. It also means he has developed competence with mathematical tools as a means of achieving some goal of understanding (Schoenfeld, 1992, p. 335). Whether or not a person develops a mathematical viewpoint depends on how much, if at all, he will reflect upon how to solve a problem (Schoenfeld, 1987).

Schoenfeld (1979, 1992) illustrates the difference in two types of learners, expert and novice, who attempt problem solving in two distinct ways. These two types of learners have developed, or have not developed, mathematical viewpoints. The expert has developed a critical viewpoint for problem solving: structure of a problem is more important than context, and critical questions are posed about the process of solving problems, especially when the problem is not routine. On the other hand, the novice problem solver who relies on intuition (if a solution feels right, it must be appropriate), may believe that if a problem cannot be solved in 10 minutes, no solution exists. This type of learner spends most of the time attempting one solution that may be completely wrong, and uses no self-checks of progress toward any explicit goal. Ericsson and Simon (1993) explained the difference in experts and novice problem solvers as understanding: "Experts often generate a representation of the problem with steps for solving it while comprehending the problem text (forward reasoning), whereas novices retrieve knowledge piece by piece using means-ends analysis" (p. xli). Researchers have referred to the differences in expert and novice learners to categorize subjects (Cobb, 1999; Goos, Galbraith, & Renshaw, 2002; Hansen, McCann, & Myers, 1985; MacClain & Cobb, 2001; Mevarech, 1999). The outward appearance of the two types of learners is apparent; the reasons for the differences are not so apparent.

Types of Knowledge

Descartes' early revelation of breaking down very complex knowledge into simpler parts in order to build an understanding is critical to the study of cognition and the human mind. Minsky (1968) agrees with this method as it applies to the study of artificial intelligence:

> To do this, one must have ways (i.e., other kinds of methods) to break a problem apart, recognize relations between the parts, solve them, and combine the results to form a solution, or at least a plan or skeleton of a solution, for the original problem (p. 15).

The complex concept of knowledge can be broken down into two types: declarative (knowing that) and procedural (knowing how). Assuming the students are learning declarative knowledge, procedural knowledge can be addressed by exploring Schoenfeld's (1992) functional categories for metacognition research questions: "1) declarative knowledge about cognitive processes, 2) self-regulatory processes, and 3) beliefs and affects and their effects on performance" (p. 347). Paying attention to these three categories supports the importance of students developing an understanding of the procedures and their meanings, a priority of most mathematics educators (p. 348).

Declarative knowledge about processes. Individuals store pieces of information in their minds and when confronted with a problem must not only recognize which pieces are relevant to the situation at hand but also access and use them to answer questions posed. Problem solvers may retrieve the information either by thinking or through interaction with the environment. Failing to solve a problem means that either the knowledge was not known—not stored in the mind—or the relevant information was overlooked—could not be accessed. Solving a problem incorrectly means some of the representations or pieces of information are imprecise, misunderstood, or just plain wrong. Declarative knowledge about processes includes intuitions, heuristics, definitions, and algorithms for solving problems along with any other type of procedural knowledge.

Self-regulatory processes. Developmental literature, artificial intelligence, and mathematics education are three domains that contributed findings to self-regulation in the 1970s and 1980s. Schoenfeld (1992) described one developmental study of four- to nine-year-olds constructing a cardboard railroad track. The older students were better at planning solutions to problems and at monitoring their progress toward the overall goal or problem solution. (This study was cross-sectional rather than following the same students through time.) When examining the increasing complexities of computer problem solving, Schoenfeld (1992) found artificial intelligence researchers used models such as Nets of Action Hierarchies (NOAH) and Opportunistic Planning Model (OPM) to plan and monitor computer appropriate tasks. Resource management was an important issue as problem-solving programs developed.

Several mathematics education researchers came to the same conclusion that "it's not just what you know; it's how, when, and whether you use it"—effective and resourceful problem solving procedures (Schoenfeld, 1992, p. 355). Lester, Garofalo, and Kroll (1989) studied seventh graders' progress in metacognitive skills to help educators teach problem solving more effectively. Schoenfeld's (1987) study of student vs. expert problem solving revealed differences in executive control, the self-regulatory part of metacognition. After explicit problem solving regulation instruction took place, students were much better at asking themselves questions that Schoenfeld posed during problem solving: "What (exactly) are you doing? (Can you describe it precisely?), Why are you doing it? (How does it fit into the solution?), [and] How does it help you? (What will you do with the outcome when you obtain it?)" (p. 206).

Beliefs and affects and their effects on performance. Understandings, feelings, and beliefs held by individuals and societies are the affective components of metacognitive skill development and effect students' mathematical conceptions and behavior. They can be categorized as: "beliefs about what is *possible*, beliefs about what is *desirable*, and beliefs about what is the best *method* for teaching mathematics"

(Schoenfeld, 1992, p. 360) and can be held by teachers, students, and society in general. Teacher beliefs tend to be generational in that they are largely influenced by personal classroom experiences prior to taking teaching methods courses. Teachers often replicate the same classroom environment which in turn shapes their students' beliefs about mathematics. "Students' beliefs shape their behavior in ways that have extraordinarily powerful (and often negative) consequences" and are shaped in large measure from their classroom experiences. (p. 359). In addition to teachers and classrooms influencing students' beliefs, the culture of the student also effects metacognitive skill development. For example, a student in the United States is more likely to believe in innate mathematical ability than a student in Japan because of culturally accepted norms. Thus, beliefs and procedural knowledge come from classroom experiences and the environment that surrounds the student.

Cognitive psychologists often turn to mathematical problem solving as an objective forum for examining declarative knowledge about processes, self-regulatory processes, and affective effects on performance (Brown, 1987; Schoenfeld, 1992; Shaughnessy, 1985; Weinert, 1987). The following section presents a proposed model for examining self-regulation, metacognition, within problem solving.

Problem Solving Framework

Metacognition has been studied in mathematics education through problem solving experiences (Lester, Garofalo, & Kroll, 1989; Schoenfeld, 1985, 1987). All cognitive processes are difficult to research, because thinking is a covert action. Self-reporting is the only method we have to assess cognitive processes, however, problems of self-reporting include the reporters changing their cognitive procedures as a result of self-reporting and giving answers they perceive the researcher wants to hear. If subjects do not recall their own cognition, they may report what they presently perceive to be true about a past situation. When self-reporting to a probability and statistics person, answers may differ from how the same person may think in the real world, outside a purely statistical domain, because students know what is expected of them through the course. Interviews may not reflect everyday thinking (Derry, Levin, Osana, Jones, & Peterson, 2000). With the increased availability of recording and, thus, verbatim transcripts as data, self-reporting may be considered explicit and objective (Ericsson & Simon, 1993) no matter what discrepancies the subject communicates. Despite the problems associated with studying covert cognitive processes, developing systematic approaches to examine the role of metacognition in specific domains is important because "metacognition instruction is most effective when it takes place in a domain-specific context" (Lester, Garofalo, & Kroll, 1989). Garofalo and Lester (1985) suggest a model that is an amalgamation of the work of Pólya, Schoenfeld, and Sternberg (p. 170).

Pólya's (1957) widely accepted phases of problem solving—understanding, planning, carrying out the plan, and looking back—identify heuristic procedures for successful problem solving, but consider metacognition only implicitly. Schoenfeld (1983) describes episodes—reading, analysis, exploration, planning, implementation, and verification—to identify transitional points where metacognitive decisions can affect success in problem solving. Sternberg (1982) outlined components of human intelligence as performance, acquisition, retention, transfer, and metacomponents. Metacomponents control all the other components and can be broken down further as "a) decision as to what the problem is, b) selection of lower order components, c) selection of one or more representations for information, d) selection of a strategy for combining lower-order components, e) decision regarding speed accuracy trade-off, and f) solution monitoring" (Garofalo & Lester, 1985, p. 170). The resulting model, shown in Table 2, "specifies key points where metacognitive decisions are likely to influence cognitive actions" (p. 171).

Table 2.

Cognitive-metacognitive framework

ORIENTATION: Strategic behavior to assess and understand a problem

- A. Comprehension strategies
- B. Analysis of information and conditions
- C. Assessment of familiarity with task
- D. Initial and subsequent representation
- E. Assessment of level of difficulty and chances of success

ORGANIZATION: Planning of behavior and choice of actions

- A. Identification of goals and subgoals
- B. Global planning
- C. Local planning (to implement global plans)

EXECUTION: Regulation of behavior to conform to plans

- A. Performance of local actions
- B. Monitoring of progress of local and global plans
- C. Trade-off decisions (e.g., speed vs. accuracy, degree of elegance)

VERIFICATION: Evaluation of decisions made and of outcomes of executed plans

- A. Evaluation of orientation and organization
  1. Adequacy of representation
  2. Adequacy of organizational decisions
  3. Consistency of local plans with global plans
  4. Consistency of global plans with goals
- B. Evaluation of execution
  1. Adequacy of performance of actions
  2. Consistency of actions with plans
  3. Consistency of local results with plans and problem conditions
  4. Consistency of final results with problem conditions

Note. From "Metacognition, Cognitive Monitoring, and Mathematical Performance," by J. Garofalo and F.K. Lester, Jr., 1985, *Journal for Research in Mathematics Education, 16*, p. 171.

The key points resemble Pólya's phases, Schoenfeld's transition points episodes, and Sternberg's subcomponents of the metacomponents of intelligence. Education researchers have chosen this framework as an organizational tool for studying students' use of metacognition (Adibnia & Putt, 1998; Mevarech, 1999; Pugalee, 2001).

Traditional mathematics classrooms have not been conducive to learning metacognitive skills (Anthony, 1996; Schoenfeld, 1985; Shaughnessy, 1992); they have typically been filled with textbooks, workbooks, and ditto sheets. There are many possible explanations for this type of classroom culture, one being that problem solving is a difficult concept to teach due to its complexities. The teacher must anticipate if a student's solution is correct and/or what the solution's implications may be. When students wander off course or struggle to make a decision, the teacher must know when to intervene. And to teach problem solving as a role model, the teacher must ideally not know the solution right away; this is very uncomfortable for most mathematics teachers. As the importance of problem solving skills is recognized and appropriate teaching strategies are incorporated in the classroom, self-monitoring and regulation should become a priority for the student. "Part of teaching problem solving involves helping to make students aware of their own metacognitive processes" (Shaughnessy, 1985, p. 407).

Elementary Probability and Statistics Problem Solving

Problem solving involves finding answers to questions when the solutions are not readily at hand, which is typical of problems in elementary probability and statistics courses. Shaughnessy (1985) found probability and statistics problems absent from problem solving research and believes this situation should change because "metacognitive aspects are brought into sharp focus in problems involving probability and statistics . . . [and] . . . stochastic problems are good examples of applied problems" (p. 409). The typical curriculum is not new mathematical concepts but is comprised of topics in elementary probability and statistics problem solving. The course becomes a problem-solving course in a statistical domain which requires previously learned mathematical knowledge. "For these reasons we believe there is much to be gained from studying the implications of probabilistic problem solving for general problem solving" (p. 410).

Elementary Probability and Statistics

When teaching students to understand what real data represent, mental processes, epistemology, pedagogy, and the relationships between them must be examined (Ainley & Pratt, 2001). Mental processes, also called intuitions, are the cognitive actions a student takes to construct meaning from data. These meanings are constructed by the individual when participating in social activities (the classroom) and reflecting on the activities (metacognitive skills) to assimilate any new information into existing cognition (Borovcnik & Peard, 1996). Intuitions about mathematics and reading are important tools when learning how to know what the data represent; therefore, educators need to address the epistemology of statistics. Students must know how to read explicit facts presented in material; read within the data by comparing the facts presented; and read beyond the facts through extension, prediction, and inference to be capable of constructing statistical intuitions.

The epistemological goal is to begin with a student's correct primary intuitions and, through instruction, extend the student's novice thinking to be more independently constructed or more expert-like. At that point secondary intuitions are the product of primary intuitions refined through social interaction and reflection. In order to develop meaningful intuitions, instruction should be concept-oriented rather than outcome-oriented with an emphasis on how the mathematical results may be used rather than on the statistical tools used to produce the result. Focusing on the rules for getting the results is unlikely to develop meaning (Campione, Brown, & Connell, 1989). Hansen, McCann, and Myers (1985) empirically found that undergraduate students who were instructed as to why to use certain formulas and algorithms to answer probability questions were more successful at solving word problems than students who were instructed in rote memorization but were less successful at recall of appropriate formulas. Onwuegbuzie's (2000) research provided evidence about graduate students' attitudes toward statistics assessments. His study revealed that students preferred "open-book" assessments because this type of assessment induced less anxiety than administering exams that allowed using limited support materials. These types of studies can provide insight in how students come to know elementary probability and statistics concepts.

When students enroll in an elementary probability and statistics course they are expected to participate in various tasks. While attending class they should listen to the instructor, participate in activities, and take exams; outside class they are expected to read the textbook and complete homework assignments. During these tasks students could experience two levels of metacognitions: an overall self-monitoring of comprehension and progress in learning stochastics (a macro level of metacognitions) and a more itemized awareness of understanding pieces of information and strategies (a micro level of metacognitions).

Micro level metacognitive situations could include the following examples. While reading the textbook, a student may realize he or she needs to read a paragraph again, because it made no sense the first time (orientation). While reading an assigned homework problem, the student may decide that using a particular formula will provide an appropriate answer to the question posed (organization). When finding the answer to an arithmetical situation, the student may decide to use a calculator which will provide a more trustworthy answer than paper and pencil (execution). And when a student arrives at the final answer to a question, he or she might pause to consider whether the answer is reasonable (verification).

In the classroom setting a student may ask the instructor to leave notes on the board a little longer when he or she realizes they must be copied down in order to be remembered for later application (orientation). When students are given the opportunity to solve problems in class, they might talk over the best plan of attack with their neighbor before writing anything down (organization). As an instructor demonstrates how to find a solution to a statistical question, a student may correct the instructor's arithmetical error (execution). And when an alternative method of solving a problem is presented in class, a student may ask "Isn't it easier just to do it the other way?" *i.e.*, "What's the point of knowing this method/concept?" (verification).

Each of the above examples is a specific point where a student must make a decision about which strategies are most viable in learning statistics and probability, micro level metacognition. They are points at which metacognitive skills help build understanding of stochastics concepts and therefore lead to learning in elementary probability and statistics courses. Pedagogy in this atmosphere concentrates on the students' construction of data and exploration into what the data may tell us rather than merely lecturing about what is represented. Guiding students to look at the data, between the data, beyond the data, and behind the data is a systematic method for examining patterns, centers, clusters, gaps, spreads, and variations. This pedagogical approach for data handling was first described by Tukey (1962) as Exploratory Data Analysis.

In his Exploratory Data Analysis (EDA) text, Tukey (1977) emphasized what others have called descriptive statistics: organizing, describing, representing, and analyzing data. He encouraged sense-making while looking for the above-mentioned patterns, centers, clusters, gaps, spreads, and variations in data. Non-parametric ordering of data and graphical representations are the methods encouraged for meaningful exploration in problem solving and reasoning. EDA differs from classical data analysis in that the main focus is on the exploration of data, not the confirmation of findings. When combined with the study of probability, EDA is a systematic study of uncertainty. Tukey's method has been compared to examining an egg to explain the characteristics of a chicken (Cobb & Moore, 1997, p. 820). He examined the characteristics of the source to describe the product.

Research

The academic subject of elementary probability and statistics is commonly referred to by researchers as stochastics. A review of the stochastics literature can be categorized into practical suggestions for teaching concepts and empirically answered instructional questions, with the former much more prevalent than the latter (Becker, 1996). Until the 1980s, stochastics had not been considered important mathematics for all students to study. Calls to reform mathematics education in primary and secondary schools drew attention to the usefulness of stochastic information for all ages (Cohen, 1995; Mathematical Association of America, 1998; National Council of Teachers of Mathematics, 1980, 1989; National Research Council, 1989). Even though many educators now consider stochastics an imperative academic subject for all students, the literature lags behind. Few researchers have reported studies which are focused on elementary probability and statistics students (Garfield & Ahlgren, 1988). This is a result of unprepared teachers, non-mainstream curriculum for stochastics, and beliefs regarding lack of importance of stochastics (Shaughnessy, 1992). As teachers and researchers recognize the need for stochastics research, the empirical studies increase in number.

The empirical stochastics literature can be categorized into research in data handling and research in probability. Both areas concentrate on common misconceptions that students develop through world experience and bring to the elementary probability and statistics classroom. Examination of these misconceptions reveals common characteristics of undergraduate stochastics students.

Undergraduate Students

Many students come to elementary probability and statistics courses with weak mathematical and reading skills (Ainley & Pratt, 2001). According to Batanero, Godino, Vallecillos, Green, and Homes (1994), "The most important factor to influence learning is the student's previous knowledge" and many statistics students lack basic knowledge

(p. 529). These weaknesses perpetuate various types of anxieties about studying stochastics. The anxieties compound, often resulting in a negative attitude that statistics is more of a hurdle to be jumped to meet career goals rather than an area of viable knowledge. A majority of the students in these courses are also of the opinion that statistics does not fit in with their professional goals, because they are in school to achieve entry into a profession other than research (Beitz & Wolf, 1997). Many students who lack confidence in their mathematics skills reported that they would not take a statistics class if given the choice, an attitude that affects the effort expended in the course (Galagedera, Woodward, & Degamboda, 2000). In a study about students' attitudes toward assessments Onwuegbuzie (2000) found statistics students' feelings, beliefs, perceptions, and metacognition often produce high anxiety about statistics classes and tests resulting in poor performance.

In addition, students bring strong intuitions to the classroom which could help in understanding stochastics, but more often cause obstacles to learning; knowledge that works in other contexts becomes a misconception in statistics. Obstacles may be ontogenic–due to child development, didactical—resulting from teaching situations, or epistemological—misunderstanding of the contextual meaning of the concept (Batanero, Godino, Vallecillos, Green, & Homes, 1994). Pre-developed obstacles make teaching stochastics to college students a difficult task, because pointing out misconceptions and explaining correct procedures often does not permanently change students' conceptions.

Conceptual knowledge and affective beliefs lead students to commit similar errors when answering stochastics questions. There is a definite pattern to the errors, which should guide educators when developing teaching strategies. What students learn depends on previous knowledge brought to the classroom, the content that is being taught, and the social environments where the learning occurs, including the tools available (Borovcnik & Peard, 1996). Teaching must override students' weak and misguided intuitions about probability and must connect to correct primary intuitions (Borovcnik & Peard, 1996). All intuitions do not necessarily need to be eliminated; they more likely need to be refined (Well, Pollatsek, & Boyce, 1990).

Stochastic Intuitions

Intuitions are immediate cognitive responses to situations. They are the cognitive pieces that allow a person to move from "I know what I am looking for" to "I know what to do" (Fischbein, 1975, p. 15) during metacognitive processes. Sometimes referred to as schemas, intuitions "select, assimilate and store everything in the experience of the individual which has been found to enhance rapidity, adaptability, and efficiency of action. Their essential characteristic in intelligent behaviour is to serve as a base for extrapolations" (Fischbein, 1975, p. 125). Extrapolations predict unknowns, and intuitions enhance the certainties of a correct prediction. The probability of a correct prediction increases with accurate intuitions.

According to Fischbein (1975) intuitions can be classified in two ways. First, there are pre-operational intuitions that are a synthesis of previous experiences relevant to the present situation, operational intuitions which follow the rules of logic presented in a situation, and post-operational intuitions which provide a diagnosis of the present situation based on previous experience. Second, they can be classified as primary and secondary intuitions, depending on whether or not formal instruction has taken place to affirm and/or refine cognitive responses. Secondary intuitions are a product of social (classroom) experience. Refining intuitions is important because "productive reasoning of any kind is achieved through heuristics, and motivated by an anticipatory approach structured as intuition" (p. 4).

Expert mathematicians employ secondary intuitions, therefore, teaching probability and statistics should include improving primary intuitions and building new, secondary intuitions; this may be even more difficult for adult students. "Once the basic cognitive schemas of intelligence have stabilized (after 16-17 years of age) modifications to the intuitive substrate seem to be difficult, if not impossible" (Fischbein, 1975, p. 12). Educators need to remember that intuitions and mathematizing are often in conflict; students' probability and statistics intuitions may be at odds with theory presented in the prescribed curriculum. "College students have had many more cognitive experiences than the young children Fischbein studied, so their primary intuitions of probability have been distorted and influenced by experiences other than instruction" (Shaughnessy, 1992, p. 486). Not knowing when or how the misconceptions develop, makes changing them a more difficult task.

Misconceptions

The misconceptions students bring to the classroom are categorized in the literature as data handling and probability. Examples of data handling concepts are mean, weighted mean, measures of spread, and regression and correlation. When learning about data handling, the algorithm for calculating mean—$\Sigma x_i/n$—is not a problem, but students typically fail to conceptually understand what the mean represents.

This leads to trouble when the value of zero is part of the data; a common misconception is that adding zero to the list of values does not change the average. And when working with weighted means, students have difficulty finding the value of an additional observation with the final mean given (Pollatsek, Lima, & Well, 1981). Instead of defining a measure of spread as how much the data differ from the measure of central tendency, students often believe a measure of spread tells how much the data differ from each other. This is a subtle yet important distinction (Loosen, Lioen, & Lacante, 1985). When studying association, linear regression and correlation, students have trouble both in choosing appropriate variables to compare and in understanding that dependence does not imply causation. This leads to additional problems in understanding that samples of populations always vary and that sample size is extremely important for representation purposes. Although the computational knowledge is known, understanding where and how it can be applied is difficult for many students (Batanero, Godino, & Vallecillos, 1994).

Well, Pollatsek, and Boyce, (1990) researched students' difficulties in understanding good sampling procedures and found statistically naïve students use inappropriate heuristics in attempting to reduce the complexities of some problems (p. 289). In an attempt to better understand the results found by Kahneman and Tversky (1982) four experiments were designed and analyzed to discover *why* students make decisions (correct and incorrect) about sampling distribution questions. Both groups of researchers found many students have misunderstandings in their interpretation of the law of large numbers. The law of large numbers says that a larger sample size is the more representative of the population than a smaller sample size because the statistics taken from larger samples vary less than statistics taken from smaller samples. Through qualitative analysis, Well et al. found most students understood that larger samples are better estimators, but they carried this understanding too far. The subjects believed that "extreme scores are more likely to occur in large samples (which is true) and that, therefore, the averages of large samples will be more variable (which is not true)" (p. 310).

Research in data handling misconceptions is sparse, however, there have been probability studies. Piaget and Inhelder (Fischbein, 1975; Shaughnessy, 1992) found that students must be in the formal operations development stage to understand probability. When children are younger than seven years old they do not know the difference between necessary and possible events and, therefore, cannot calculate ratios. Between seven and fourteen the students know the difference but need more mathematics maturity and understanding of combinatorics to make abstract models necessary for understanding probability. It isn't until age fourteen that students can conceptually understand ratios/probability counts. Fischbein disagrees with these results saying children in the concrete stage (seven and younger) can learn about probability if given appropriate instruction. Children are more open to important concepts in probability, because they have not experienced as many social occurrences to develop common and robust misconceptions (Shaughnessy, 1992).

Three main types of probability misconceptions are called representativeness, availability, and adjustment and anchoring. Representativeness demonstrates a lack of understanding of sampling. The "belief in the law of small numbers" (Tversky &

Kahneman, 1971) reveals a common belief that sample size is not important when determining probability. The following problem has been given to many people.

> A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50% of all babies are boys. The exact percentage of boys, however, varies from day to day. Sometimes it may be higher than 50%, sometimes lower. For a period of one year, each hospital recorded the days on which more/less than 60% of the babies born were boys. Which hospital do you think recorded more such days? a) the larger hospital, b) the smaller hospital, c) about the same (i.e., within 5% of each other) (Well, Pollatsek, Boyce, 1990, p. 290; Shaughnessy, 1985, p. 401)

College students very often neglect effect of sample size and choose c (when the correct answer is b) revealing a belief that sample size does not affect variability.

The “gambler’s fallacy,” another misconception of representativeness (Batanero, Godino, Vallecillos, Green & Homes, 1994) says each outcome will return the overall occurrence of an event closer to the theoretical probability. Many people believe that after a long run of one outcome it is more probable that a different outcome will occur. For example, a baseball player with a good batting average has not had a hit for a long time—he is “due” for a hit. Also, if a coin tossed in the air (0.5 probability of heads) has landed on heads many times, a tail is more likely to occur. Both examples show a belief that the next event will help to adjust the sample’s frequency closer to the theoretical frequency. The believer of gambler's fallacy interprets probability as likelihood for each event rather than an overall likelihood among all the observations.

The misconception of availability means people tend to believe an event is more likely to occur if they have personally experienced that event. For example, if someone has recently been divorced or knows many people who have divorced, they might believe the overall divorce rate to be higher than it really is. Or if a driver has been in an accident in a particular town or at a particular intersection, he might believe the accident rate at that location is higher than other places. In addition, people tend to think of their unique experiences as more surprising than the same unique experiences happening to others (Shaughnessy, 1992).

In anchoring misconceptions, students and experts often believe a conditional probability is larger than the parent event; this misconception is called "conjunction fallacy." For example, when asked for the probability that someone is older than 55 and had a heart attack compared to the probability that someone had a heart attack, many students believed the first probability to be higher (Kahneman, Slovic, & Tversky, 1982). The conjunction fallacy leads to difficulty understanding conditional relative frequency tables.

Pfannkuch and Brown (1996) found many misconceptions in a study about omnipresence of variation in sampling. In interviews, their adult subjects expressed belief in gambler's fallacy, availability bias, and the inappropriate belief in "the law of small numbers."

Conclusion

Previous research has led to many contributions in improving mathematics education yet some students are still being left behind. While educators strive to guide students to master concepts, theories have been identified to make the task possible. Philosophers across the centuries considered thinking about how students learn in general. They pointed toward examining the processes necessary for showing people how to take charge of their own learning to develop into expert problem solvers. Constructivism is one of the accepted theories and is especially pertinent to guiding students to take control of their own learning. If students learn metacognitive skills, they develop learning strategies to monitor their construction of knowledge, including mathematical problem solving which is a basis for statistical problem solving.

Some researchers have studied metacognition within mathematical problem solving, one step away from statistical problem solving. Schoenfeld has reported that students who are explicitly instructed to ask questions that monitor progress in problem solving exhibit expert problem solving behavior. Garofalo and Lester developed a systematic procedure for identifying metacognitive thoughts. Shaughnessy reported instruction in using metacognitive skills improves success rate in problem solving. However, he found probability and statistics problems absent from problem solving research. In Shaughnessy’s (1985) words, this situation should change because “metacognitive aspects are brought into sharp focus in problems involving probability and statistics . . . [and] . . . stochastic problems are good examples of applied problems” (p. 409).

As probability and statistics is given increased importance in mathematics education, research about the students' common misconceptions indicates a need for going beyond purely cognitive solutions to increased success. In an effort to disprove Piaget's developmental theory regarding children learning probability, Fishbein argued that through life experiences college students have developed misinformation about probability and therefore young children were more accepting to instruction in combinatorics. Kahneman and Tversky concluded that students' misconceptions about the importance of sample size distorted their interpretations of the Law of Large Numbers. Well, Pollatsek, and Boyce used qualitative research methods to further explore students' interpretations and found students often solve statistical problems correctly but for the wrong reasons. Other studies connect affective issues to success in learning stochastics; there is still a gap connecting metacognition to the domain of probability and statistics.

To develop mathematical power for more students, as reform demands, we need to examine students' conceptions and misconceptions as clues to what happens in cognitive processes. Students make common errors that are strongly ingrained in their minds as correct. Explaining the correct concepts and helping the students build statistically appropriate schemas does not always ensure the correct concepts will be employed outside the classroom. In addition, decisions evaluated as correct are sometimes made for inappropriate reasons. What appears on the surface to be correct may be a result of incorrect thinking when studied more in depth; students' verbal explanations may show what role metacognition plays.

Although studying students' metacognitive skills while learning elementary probability and statistics concepts is a somewhat complicated task, it can and should be done. The constructivist theory of learning provides a basic foundation for examining thought processes in any domain; it is especially relevant to what students think about while assimilating statistical knowledge into what they already know as true for problem solving. Meaning is constructed through reflection and metacognition is specific thought about one's own cognitive processes and how well progress is being made in solving problems. Researchers have found empirical evidence that metacognition is a viable part of learning. Math educators are searching for strategies to nurture students' problem solving skills. What role does metacognition play in constructing meaning about probability and statistics concepts?

In order to determine if students are using micro level metacognitive skills to learn stochastics, qualitative methods of data collection were utilized. The following study was designed to examine evidence of students' use of metacognition while learning stochastics concepts and to answer guiding questions:

1. What are the students doing cognitively to learn concepts in elementary probability and statistics?
2. What role does metacognition play when students are learning how to make decisions that require an understanding of probability and statistics concepts?
3. Is metacognition a necessary element for success in undergraduate elementary probability and statistics courses?

Chapter Three

Methodology

The focus of this study was on statistics students' use of metacognition. Students were observed in their classroom and individual profiles were developed through qualitative research methods. Evidence of metacognitive skills were observed both in the classroom and in problem-solving situations.

Participants

Forty-nine of the 50 students enrolled in one section of the third quarter of a three-quarter sequence Elementary Probability and Statistics course offered by the mathematics department of the arts and sciences college at a large midwestern university were the human subjects observed for indications of using metacognition while learning elementary probability and statistics concepts. One of the students enrolled in this section was under the age of 18; classroom observations did not include his behavior and he was not considered for out-of-class participation. All the other students who attended the class signed the letter of consent and were included in classroom observations. These spring quarter statistics students had completed the first two quarters of the three-quarter sequence of Elementary Probability and Statistics, either immediately preceding the quarter in which they were observed or at some other time. Although enrollment in the class is often a requirement to degree programs, some students also enroll because they plan to attend graduate school.

Information about gender, race, and class rank, collected through classroom observation and the class list, is summarized in Table 3. Twenty-nine of the forty-nine observed students were female and most of them were white, traditional, young adult students (between the ages of 18 and 25). Many of the students were seniors, however, some were juniors and sophomores, and only five were freshmen.

Table 3.

Summary of participants' gender, race, and class rank

	Whole Class	Selected Seven Students
Gender		
Female	29 (59.2%)	6 (85.7%)
Male	20 (40.8%)	1 (14.3%)
Race		
White	40 (81.6%)	6 (85.7%)
African American	6 (12.2%)	1 (14.3%)
Eastern Indian	3 (6.1%)	0 (0%)
Class Rank		
Seniors	19 (38.8%)	2 (28.6%)
Juniors	13 (26.5%)	3 (42.9%)
Sophomores	12 (24.5%)	2 (28.6%)
Freshmen	5 (10.2%)	0 (0%)

The seven students who agreed to participate in the out-of-class interviews included one white male (senior), one African American female (junior), and five white females (one senior, two juniors, and two sophomores). The distribution of the sample of seven students who participated outside the classroom is similar to the whole class for race and class rank but differs slightly in gender. The selection of the sample was based on criteria other than gender, and statistically some variation from the class distribution is to be expected.

Instruments and Forms

Consent form. Students were informed, orally and in writing, of the purpose of this study, their option to participate or not, and a method for contacting the researcher and the researcher's advisors. The written information was presented in the Consent Form (see Appendix A) that each student was asked to sign. The Consent Form further explained that the students' responses to questions would not affect their statistics course, and that participation would not add to nor take away grade points. A section of the Consent Form explained an opportunity for the students to earn a modest stipend by participating in individual and group interview sessions held outside the classroom, however, no other type of compensation nor reward was given to those who elected to participate. The students were assured orally and in writing that the "course will not be different from other sections of this statistics course because of this study."

Classroom organizer of students' indications of metacognition. The classroom observation form, (see Appendix B), served as an organizer of students' indications of metacognitions during class sessions. Its purpose was connecting recorded classroom observations to Garofalo and Lester's (1985) cognitive-metacognitive framework described previously beginning on page 29. The four main headings in the framework—orientation, organization, execution, and verification—were listed in the organizer with space for quoting and/or describing classroom instances of evidence of metacognition described within the framework's categories. A fifth category, "Lack," was created by the researcher and added to the Classroom Organizer to provide space for instances where metacognition might have been appropriately used by a student but in fact was not. Examples of how this instrument was used are in Appendix C.

Data analysis problem for individual interviews. The data analysis problem in Appendix D was given to the seven individually interviewed students. (The Appendix includes solutions to the questions posed.) The chosen problem was a balanced combination of *recalling* (calculating and interpreting descriptive measures) and *extending* (how descriptive measures are used to interpret basic stock market analysis) previously discussed statistical concepts. The participant was asked to write answers to the questions on the same sheet of paper as he or she worked through the problem out loud. The main purpose of the interview was to interact with and observe students reflecting on and verbalizing their thought processes about specific statistical concepts during problem solving.

Data analysis problem for group interviews. The group problem (see Appendix E) was an edited version of a similar type problem presented during class instruction. (The Appendix includes solutions to the questions posed.) The purpose of this instrument was to make visible evidence of otherwise invisible metacognitive processes in the students' conversation while they solved a somewhat familiar problem together. It was an activity that recreated metacognitive thought processes experienced in the classroom to learn probability and statistics strategies, but was given in an environment that could be video and audio-taped for analysis purposes. The problem was given in two collaborative group sessions: four students in one group and three students in the other group. Each student was given a blue personal copy for ease in reading, and each group was asked to write consensual answers to the questions on one white copy.

Other instruments. In order to analyze the responses of the students all interviews were recorded on an audio-tape cassette; the group interviews were also videotaped. The data were transcribed in Microsoft Word and entered into a Microsoft Excel spreadsheet. The spreadsheet was used as a tool for organizing the data, coding it by categories in the metacognitive framework, and sorting evidence identified.

Procedure

The following section describes in detail what took place during the data collection process. All of the students taking the course were introduced to the study, asked to sign a letter of consent and observed during eleven of the class meetings. After the first midterm exam, ten students were asked to participate outside the classroom. Only seven of the invited ten students responded positively and participated in two types of self-reporting: retrospective and concurrent (Ericcson & Simon, 1993, p. 16). They were interviewed about one day of class notes taken (retrospective), solved a statistical problem alone (concurrent), and solved another statistical problem with a group of students (concurrent).

Students' introduction to this study. The participants were informed that the purpose of this study was to become familiar with the thinking processes of students enrolled in statistics. In order to thoroughly analyze the meaning of students' thoughts, they would be observed in class and perhaps be asked to participate in interview sessions for which they would receive a modest stipend. The participants were also informed that the researcher would be given access to their first midterm exam grades for this class and their spring quarter course grades. Confidentiality was guaranteed, and the students were assured that any reported data would not be traceable to individuals. Key points explained to the class were: letter of consent, classroom observations, class notes review, and individual and group problem solving sessions.

Letter of consent. During the study introduction given during the fourth meeting of the course, the students were told that the university required their written permission to be observed and interviewed. The entire class was asked to sign and return a letter of consent, mentioned above, as evidence of their agreeing to be a part of the study. Participation was not a course requirement. Students who agreed to participate did not receive any compensation or reward, but those who signed the letter and subsequently attended two individual interviews and one group session received a small monetary incentive of five dollars per out-of-class session. Any student who agreed to be a part of the study at the beginning of the course always had the option of withdrawing his or her consent, with absolutely no adverse consequences. The letter provided the researcher's email address in case questions or comments arose during the quarter; an advisor's and a department head's phone numbers were given to the students in the letter of consent in case any concerns were not sufficiently addressed by the researcher.

Because not all the students were present in the classroom during the introduction, it was repeated during the following class and repeated a third time individually to students who had not yet returned a signed consent form. Forty-nine of the students signed and returned the form after hearing the introduction to the study. Students who were enrolled in the class and had not signed the form after the third explanation did not return to class and were not part of the study.

Classroom observations. After collecting consent forms from each student, the classes for the rest of the quarter (Tuesdays and Thursdays from 11:00 a.m. until 12:15

p.m.) were observed and notes on actions and comments the students made were written. After each class, the day's data were transferred to a Classroom Organizer designated for that day (see Appendix C), categorically recording all student behavior and comments that indicated metacognitive thinking and any instances in which metacognition may have been useful for understanding but were not apparent.

Student selection for further data collection. At the time the described plan was decided, it was believed, by the researcher, that more successful students would use metacognitive skills in learning probability and statistics. The proposed selection process was meant to provide a cooperative mix of metacognitive and non-metacognitive thinkers. After the first midterm exam for the course was administered and returned, the students' test scores were arranged from lowest to highest and divided into quartiles. Through purposeful sampling, ten students—five near the first quartile and five near the third quartile—were asked to participate in each of three types of activities outside the class—individual class notes review and individual and group problem solving. This method of selection was an effort to eliminate the top and bottom performers in the class from the rest of this study to examine similar types of students rather than include extreme performers and non-performers.

Originally, only six students were to be invited to participate outside the classroom. Because some students were willing to be observed in the class but did not wish to spend additional time involved with the study, the selection procedure was altered slightly to assure *at least* six students participated outside the classroom. Instead of three students from the first quartile and three students from the third quartile, five students near each quartile were asked to participate further.

Class notes reviews. Of the ten students asked to participate outside the classroom, eight agreed to schedule a time to share their class notes with the researcher in her office. In order to accommodate the students' schedules and to complete all eight class notes reviews in a timely manner, two students agreed to wait 20-30 minutes for another student to finish her interview after the class was over to review their notes with the researcher. Therefore, all eight class notes reviews were conducted on May 1 (2 students), May 6, May 13, May 15, May 20, and May 22 (2 students). Subsequently, the second student who participated in the class notes interview on May 22 did not follow through with completing her individual problem solving interview nor with either of the group sessions; this participant was dropped from the outside-of-class segment of the study.

The individual class notes review sessions consisted of the researcher and student reading his or her notes taken during the class period immediately preceding the interview. By walking through the notes immediately after the class, the student being interviewed was able to remember what he or she was thinking at the time in order to explain thought processes that were occurring during the class. For approximately 20 minutes, the researcher read and asked each student questions such as "Why did you write this down?" and "What were you thinking here?" and "How well were you understanding this?" The questions were directed toward revealing specific, micro level metacognitive thoughts the student might have had in the classroom. Copies of the students' notes were made, and all of the review interviews were audio-taped and transcribed for analysis purposes.

Individual problem solving sessions. After each student completed the class notes review, he or she scheduled a time to meet with the researcher to solve one multi-step statistical problem. These problem-solving interviews took approximately 30 – 45 minutes for each participant and were scheduled on May 15, May 19, May 21 (3 students), and May 23 (2 students). All seven of the students who completed the individual problem solving interview subsequently completed the third interview as well and were included in the out-of-class part of the study.

This interview consisted of the participant solving, out loud, the statistical problem found in Appendix D. Oral directions were: "As you solve this problem, please express out loud all your thoughts, attempts, mistakes, and re-starts. I am interested in your thinking as you attempt to solve this statistical problem. You may use any of the items in front of you including the calculator, paper and pencil, and the statistics course textbook (Moore & McCabe, 2003). The process of your thinking is what I'm interested in, not whether you solve it correctly." During the interview, questions such as "What do you find difficult?" and "How do you deal with that type of problem?" elicited specific, micro level metacognitive thoughts the student might have been using in statistical problem solving. All seven of the individual interviews were audio-taped and transcribed for analysis purposes.

Group problem solving sessions. Both group problem solving sessions were held on May 27 in a classroom near the room in which the course meetings were held. The original plan for the group problem solving session was to have all the students who participated in the individual interviews meet at one time to solve a statistical problem together. If all of the students did not come to the session, data were to be collected on those students who did, and if fewer than three students came to the session, it was to be rescheduled for another time. This plan was altered slightly because available space was not adequate for seven people. Instead the students were recorded in two groups solving the same problem for approximately 30 – 50 minutes each. The first four students to arrive formed Group A, while those coming somewhat later became Group B. Group B waited in the hall until the first group left and then participated in their group session, answering the questions to the same problem that Group A solved.

In each session, the students were seated at a table and were given a personal copy of the problem for ease in reading (see Appendix E). Each group was given one additional copy of the problem for the participants to record their agreed-upon answers to the multiple steps of the problem. The directions given to each group were "Solve this statistical problem together as a group. There is a blue copy for each of you and a white copy for writing your final agreed-upon answers. Make sure you agree with each other and try to speak out loud what you are thinking and what you want the others to know. You are welcome to use the textbook, any calculator, your notes, or whatever you would like to use—no restrictions." During the group sessions, the researcher acted as a guide, only answering students' questions and providing prompts when a group was "stuck" for too long of a time. Both group sessions were audio and videotaped and transcribed for analysis purposes.

Data Analysis

Patton (1990) states "A multimethod, triangulation approach to fieldwork increases both the validity and the reliability of evaluation data" (p. 245). He suggests using a combination of observations and interviews as data sources for validating findings. After observing the Elementary Probability and Statistics classes, discussing class notes, and interviewing the students, individually and as a group, all data were transcribed and descriptively coded according to the categories in Garofalo and Lester's (1985) metacognitive framework. As suggested by Miles & Huberman (1994), the data were examined across data sources (students), methods (interview types), and data types (cognitive-metacognitive framework categories) in order to triangulate patterns of metacognition use and success in learning probability and statistics according to the literature. Different methods of data collection revealed reliable, common evidence of the participants' use of metacognition in their processes of understanding stochastics.

The classroom observation notes were entered into the Classroom Organizer (Appendix B) according to the categories identified in Garofalo and Lester's (1985) cognitive-metacognitive framework—orientation, organization, execution, and verification—and the researcher's category called lack. The four metacognition categories are a combination of Pólya's phases, Schoenfeld's episodes, and Sternberg's components of intelligence into specific mental actions that aid in solving problems (p. 171). The results are presented by each of the framework's categories of metacognition and the researcher's additional category of lack of metacognition.

The data from the three types of interviews were entered in an Excel spreadsheet and coded according to the same cognitive-metacognitive framework. As the data were analyzed and coded, four themes found in the literature became apparent: novice vs. expert problem solvers, statistics as a viable subject, self-reporting, and the cognitive-metacognitive framework. The themes pointed toward the answers to this study's guiding questions: 1) What are the students doing cognitively to learn concepts in elementary probability and statistics? 2) What role does metacognition play when students are learning how to make decisions that require an understanding of probability and statistics concepts? and 3) Is metacognition a necessary element for success in undergraduate elementary probability and statistics courses?

When it was observed that all the students contributed evidence regarding each of these themes, individual profiles were developed in a way that organized the students' thoughts across the data. Each student's profile begins with a brief introduction that identifies overall, personal and academic characteristics about the student. The exam score that was used for selection purposes and the final course grade for the interviewed student were included in the introduction as an indicator of level of success in learning statistics. Following the introduction for each student, evidence from the coded data for the four themes is presented. This combined information provided a foundation for discovering the role of students' use of metacognition while learning stochastics, a critical step in understanding why some students are successful in learning probability and statistics and other students are not considered successful.

Chapter Four

Results

The following chapter presents the results of this study in two sections. The first section, Classroom Observations, categorizes observed student behaviors and comments in the classroom according to Garofalo and Lester's (1985) cognitive-metacognitive framework. Examples taken directly from the data supporting the results are followed by the date of the Classroom Organizer in parentheses. For example (4-15-03) indicates a student behavior or comment observed in class on April 15, 2003.

The second section, Out-of-class Interviews, presents a profile of each student who participated in the interviews. Data supporting these results are followed by a citation in the form of pseudonym, type of interview (CN denotes class notes interview, IPS denotes individual problem solving interview, and GPSA or GPSB denotes the appropriate group problem solving session for Group A or Group B, respectively), and line number within the transcript. For example, (JessicaCN, 1-5) identifies a quotation from Jessica's class notes interview transcript, lines 1 through 5. The ordering of the students in the second section is by group in which the student participated, Group A first and then Group B, and within each group the first profile is the student who scored higher than the other students in the same group on the first midterm exam. The second student presented scored the next highest on the midterm exam within his or her group problem solving interview and so on. Group A consists of four students: Jessica, Amanda, Natalie, and Cathy, in that order. Group B is Maggie, Charlene, and Mark, in that order. Each student's profile is organized according to four themes that emerged during data analysis: novice vs. expert problem solving, statistics as a viable subject, self-reporting, and Garofalo and Lester's cognitive-metacognitive framework. The final paragraph of each profile is a summary of the student's metacognitive thinking.

Classroom Observations

The purpose of the classroom observations was two-fold: the researcher and the students became acquainted with each other in preparation for the out-of-class interviews and the researcher observed student behaviors that provided evidence of a student, or students, using metacognition while learning elementary probability and statistics during the quarter. By observing the classes, taking notes on evidences of metacognition used in the classroom, and classifying the students' behavior and comments according to Garofalo and Lester's (1985) metacognitive framework categories, evidence about use of metacognition was organized.

Comments and behaviors were categorized as Orientation if it appeared that a student or students were working toward comprehending the problem at hand, analyzing given information, identifying previous similar situations, representing given facts, and/or assessing the possibility of success. Organization was the code applied if the comment or behavior indicated the student was identifying goals or subgoals for the plan to solving the given problem. If the student gave evidence of performing steps toward solving, or evaluating benefits of processes, this was considered Execution. If the behavior or comment was evaluating adequacy or consistency of the results for the first three categories, it was considered evidence of Verification.

Eleven of the 20 classes were observed because three classes took place before IRB approval was granted, one class was cancelled, and five classes were individual, written assessment classes that were not conducive to collecting data. Immediately following all classroom observations, notes taken during the class were transferred to a copy of the Classroom Organizer (Appendix C) for analysis. The classification process revealed the following ideas by the framework categories.

Orientation

The instructor and some of the students spent a lot of class time developing an understanding of the problems presented. Some of the evidence in this category included the following: "Students copying information from the board" (4-10-03); "Teacher said, 'Look in your table to see if it agrees.' A few people started looking in the book for the table" (4-15-03); "Student is sitting forward in his seat (to better hear/understand?)" (4-17-03); "most students taking notes about left-handed and right-handed batters" (4-24-03); and "One student who hasn't answered any questions answered when I moved behind him. Another previously silent student answered a question after I moved here" (4-29-03); and "Active participation, more students sharing with each other" (5-20-03).

As the instructor guided the students in working through many examples, questions were posed and discussed by both the instructor and some of the students. "Student asked 'How do you know if we use a 95% confidence interval?'" (4-10-03); "Two students whispering as Teacher and students work through problem" (4-15-03); and "Jessica and girl in front row are responding to Teacher's questions. They answer many of Teacher's rhetorical questions. Do they need to do this to keep focused on learning?" (4-17-03). It appeared that the students who sat in the middle, near the front of the classroom participated more than the students who sat in other locations. "One student answering calculation questions. He sits in the first row in the middle when he is in class" (4-14-03); and "My personal observation is students who sit in middle front communicate with Teacher more. Is this metacognition at work?" (4-29-03).

Organization

The only form of students developing a plan to solve the problems in the classroom appeared to be the students retrieving and using their calculators. "Many students pulled out TI83 calculators when asked to calculate the confidence interval" (4-10-03); and "Teacher directed students to find xbar of 7 numbers on board. A few students took out calculators" (4-24-03). During one class it was noted that one student asked "What would be the smart thing to do if you got this question on a test?" (5-1-03) which was a form of developing a plan for solving problems during an individual assessment. The instructor presented various examples of the statistical theories and worked through the problems with guiding questions for the class to follow. Few chances were given for the students to develop a plan before the steps were executed to solve the problem.

Execution

Some of the students consistently participated in the execution of solving the problems presented in class. They answered the instructor's questions about solutions to different steps within the multi-step problems. "Teacher assigned: Find 95% confidence interval for x = 32 and n = 95. One student found answer and replied '(.242, .432).' Teacher asked 'Does anyone agree with that?' Several students said 'Yeah'" (4-10-03);

and "I hear whistles from students when they found χ^2 on TI83. It's so much easier than doing all the calculations" (4-29-03). The students who participated, presented evidence of execution by writing in their notebooks, punching buttons on their calculators, and by communicating with the instructor and other students when they found statistical/ algorithmic answers. Examples from this category were "Most students working problems" (4-15-03); "Teacher gave problem with a 3x4 matrix. Most students working with calculators" (4-29-03); and "Teacher asked 'Which selection is it?' Student responded '6' . . . Teacher asked 'What z value do you see?' Student '-2.447' Teacher 'and the p value?' Student said '.014'" (4-17-03).

Verification

The instructor and the students gave evidence of verifying calculated answers. However, most of the verification was done by agreeing on the calculated answer, rather than addressing reasonableness within the problem context. "Student commented politely 'Wouldn't it be 54?' (instead of the 52 that Teacher wrote on the board)" (4-10-03); "Teacher asked, 'How many think this is right? [pause] How many think this is wrong?' Not much response from the students. Teacher continued 'How many are not thinking?' Several students raised their hands. Teacher repeated the questioning and more students participated in the vote. (lack of metacognition? When behavior was pointed out, students cooperated/participated more.)" (4-15-03); "Student found error written on board—an answer given by another student 5 minutes earlier" (4-29-03); and "Student asked other student why her calculator showed a completely different answer. Other student found her input error" (5-6-03). Some discussion about the meaning of the answers did take place. There were only a few examples of verifying the answer made statistical sense: "Teacher said, 'I goofed up here. Do you understand how this works?' One student had a definite 'Yes.'" (4-15-03); "Student turns to another student and says 'If in critical region then reject H0' Other student responds 'Yeah, I think so.'" (4-17-03); "Teacher asked 'Do you notice a pattern?'" (5-13-03); and "Mark answered Teacher's question 'What's another way we'll get a large quotient for F?' with 'A small denominator' fairly loudly" (5-20-03). These comments indicated an understanding of the calculated answer was important to learning the theory.

Lack

Many students did not participate in the course communications. They studied other subjects, slept, chatted, played games, or left before the class was over. "student sleeping" (4-10-03); "only a few students answer Teacher's questions. The same few keep nodding" (4-29-03); and "One student reading magazine, taking notes now" (5-15-03); "One student just staring—no notes, no calculator. I don't know his name but he was studying Spanish one day. He's reading something (book) now" (5-15-03); and "one student working on calculator as Teacher works on board with formulas, 5 minutes later playing a game on TI83" (4-17-03). Lack of participation during one class resulted in the Classroom Organizer note "Do these students understand or are they so lost they are dumbfounded?" (5-20-03).

Summary

Attending classes provided the opportunity for the researcher and the students to become comfortable with one another. Some of the students expressed an interest in the study, and others made it clear they could not participate outside the class. Most of the students selected by the described process happened to be those who attended class and were available for further discussions.

It appeared that the instructor's main purpose in class was to provide examples grounded in statistical theories presented. He and the students worked through many problems together. This was a benefit for the students who participated, but many more students found other things to do in class or did not attend at all. (Through rough comparison of assessment days and instructional days, attendance was approximately 60% on lecture days, with many of the students leaving class before the instructor was finished.)

Out-of-class Interviews

As described on page 53, the original plan for this study was to interview six students outside the classroom in three different types of interviews: class notes review, individual problem solving, and group problem solving. The students proposed to be invited to participate outside the class were selected based on the first midterm assessment: three students near the first quartile and three students near the third quartile. Because it became evident in the classroom observations that some students did not want to participate outside the classroom, five students near each quartile (a total of 10 students) were asked to participate. Of the five students near the first quartile, one student replied that he was too busy and the other four agreed to participate. As for the five students near the third quartile, one student did not respond and did not attend class after signing the consent form. Another student from the third quartile participated in the class notes review but failed to attend both the individual and group problem solving interviews; she was dropped from this part of the study. The other three students selected from the third quartile participated in all three interviews for a total of seven interviewed students.

The out-of-class interviews were recorded, transcribed, and entered into an electronic spreadsheet. As the data were coded according to the categories in Garofalo and Lester's (1985) metacognitive framework, evidence of otherwise invisible metacognitive processes became visible in the students' conversations while they discussed class notes and solved somewhat familiar problems individually and in a group.

Further analysis of the data resulted in identifying four literature-related themes in the data: novice vs. expert behavior (Schoenfeld, 1978), statistics as a viable subject (Greer, 1996), self-reporting (Ericsson & Simon, 1993), and Garofalo and Lester's (1985) cognitive-metacognitive framework. A student was considered expert when he or she paid attention to the structure of a problem and posed critical questions while working through novel situations. A novice relied on intuition and feelings, rarely monitoring progress toward an appropriate solution (Schoenfeld, 1987). Indications that a student considered statistics to be a viable subject included the student identifying areas outside the course of Elementary Probability and Statistics in which statistics applied, such as polls and history courses. In contrast, if the student did not consider statistics related to real world applications or expressed a lack of interest in learning the concepts, he or she did not connect and/or recognize the potential usefulness of the concepts presented in the course. There was no reason to use or develop an understanding of probability and statistics concepts. Despite the problems identified with self-reporting, triangulation of multiple data types revealed various levels of awareness of and ability to report cognitive processes. “The difference between experts and advanced beginners is often not in whether they have the necessary knowledge in memory but in whether they can access it reliably when it is needed” (Ericcson & Simon, 1993, p. xli). Both accessing information and verbalizing thoughts aloud contributed to the students’ evidence of veridical self-reporting. The fourth theme, the cognitive-metacognitive framework, provided the codes used in this study. Tallies of the students’ evidences of metacognition are displayed in Table 4. Transcription page counts for class notes and individual problem solving interviews are shown in Table 5. The numbers in Table 5 present an estimate of how much the students were willing to share thoughts with the interviewer, comparatively. During coding and data analysis, each of the four themes appeared somewhere in all seven of the students’ transcripts.

Following brief introductions for each student, metacognitive contributions are described according to the four themes. The order of the students’ profiles is by the group problem solving session (Group A then Group B) in which the student participated. The first four students who arrived at the location on the day of the interview, were in Group A, and the remaining three were in Group B. Within each group the students’ descriptions are in order of their first midterm score, highest score to lowest. The first

Table 4.

Tallies of students' evidence of metacognition

	Group A												Group B								
	Jessica			Amanda			Natalie			Cathy			Maggie			Charlene			Mark		
	N	I	G	N	I	G	N	I	G	N	I	G	N	I	G	N	I	G	N	I	G
Orientation Total	**10**	**9**	**4**	**3**	**8**	**0**	**6**	**6**	**6**	**4**	**2**	**7**	**6**	**8**	**5**	**1**	**7**	**20**	**4**	**10**	**13**
Comprehension	1	3	2	1	0	0	3	2	1	3	1	1	4	1	1	1	2	2	1	4	1
Analysis of Information	3	0	1	1	3	0	1	2	1	1	1	2	1	4	3	0	3	14	1	4	9
Familiarity	2	3	1	0	3	0	1	2	1	0	0	4	0	2	1	0	1	4	2	2	3
Representation	4	0	0	1	1	0	1	0	3	0	0	0	1	0	0	0	0	0	0	0	0
Difficulty/Success	0	3	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
Organization Total	**4**	**8**	**3**	**3**	**3**	**1**	**6**	**1**	**6**	**2**	**0**	**5**	**4**	**2**	**8**	**4**	**6**	**2**	**3**	**3**	**6**
Goals/Subgoals	1	3	2	0	2	0	0	1	1	0	0	1	1	0	2	0	2	0	0	1	5
Global Plan	2	3	1	1	1	0	5	0	0	1	0	2	0	0	2	4	0	0	3	1	1
Local plan	1	2	0	2	0	1	1	0	5	1	0	2	3	2	4	0	4	2	0	1	0
Execution Total	**0**	**3**	**3**	**0**	**2**	**0**	**0**	**2**	**2**	**1**	**1**	**1**	**0**	**1**	**3**	**0**	**2**	**3**	**0**	**3**	**1**
Performance	0	2	2	0	2	0	0	2	2	1	1	1	0	1	3	0	2	3	0	1	1
Monitor Progress	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Trade-off Decisions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
Verification Total	**1**	**4**	**4**	**0**	**2**	**4**	**0**	**2**	**1**	**0**	**1**	**3**	**1**	**4**	**10**	**0**	**2**	**5**	**0**	**1**	**15**
Orientation/Organization	1	1	2	0	0	2	0	0	1	0	0	2	1	0	2	0	0	1	0	0	4
Execution	0	3	2	0	2	2	0	2	0	0	1	1	0	4	8	0	2	4	0	1	11
Total by Interview	**15**	**24**	**14**	**6**	**15**	**5**	**12**	**11**	**15**	**7**	**4**	**16**	**11**	**15**	**26**	**5**	**17**	**30**	**7**	**17**	**35**
Lack	**0**	**0**	**1**	**1**	**4**	**6**	**1**	**7**	**2**	**10**	**6**	**0**	**0**	**1**	**1**	**1**	**1**	**0**	**0**	**3**	**2**
Grand Total (excluding lack)		**53**			**26**			**38**			**27**			**52**			**52**			**59**	

* "N" indicates Class Notes Session
"I" indicates Individual Problem Solving Session
"G" indicates Group Problem Solving Session

Table 5.

Counts of transcript pages for individual interviews

	Midterm Quartile	Class Notes	Individual
Group A			
Jessica	1	13	15
Amanda	1	4	12
Natalie	3	12	12
Cathy	3	8	9
Group B			
Maggie	1	9	14
Charlene	1	5	16
Mark	3	10	10

group of students, in order of their descriptions, were identified by the pseudonyms Jessica, Amanda, Natalie, and Cathy. The students in the second group were called Maggie, Charlene, and Mark and are presented in that order.

Jessica

Jessica's midterm exam score of 88.6% placed her very near the first quartile (85.4%). Her final course grade was a B+. This student appeared to be very confident about her learning abilities, yet she showed gaps in understanding statistical concepts. Jessica was a very talkative person which resulted in 13 and 15 pages of class notes and individual problem solving interviews, respectively (see Table 5), the greatest number of pages of transcripts in the both groups of students. During the class notes interview she complained about statistics problems being too wordy and used this as an excuse for not doing homework: ". . . I was just getting bogged down with words, in the book problems" (JessicaIPS, 190).

Because of life experiences Jessica expressed a determination to do what it takes to graduate. She takes things apart to accomplish goals and this continued in problem solving skills in statistics. She was aware that many students do not want to do what it takes to be successful in school, but because she is willing to do the required work, Jessica believed she understood statistics and could use it in life. Her comprehension was not completely accurate, but awareness of her own and others' learning is a potentially useful tool for further development.

Novice vs. expert behavior. Jessica's behavior was expert rather than novice. She understood that in order to learn, she needed to break large problems or concepts down into workable pieces. Jessica paid attention to the structure of the problem and could relate this to many areas of life. For example, she related mathematical problem solving to what she knew about learning a foreign language:

> If you don't learn how to con, if you don't learn your verbs, and you don't learn how to conjugate them, then it's going to be helpless when you're doing more advanced, you know, learning and stuff like that. I mean, everything has a basis and it's like, you have to understand the basic things, otherwise when you learn the advanced stuff it's useless (JessicaIPS, 166).

She expressed the need to "memorize" certain basic facts, but it was important to Jessica to be able to understand how everything fit together. "But if you don't know what it's for, you can memorize whatever you want. But if you don't know how to apply it, it's useless" (JessicaCN, 198).

For the most part, Jessica demonstrated understanding but failed to correctly explain slope, which is a fairly simple but important concept in linear regression. Jessica was exercising metacognitive strategy toward orientation yet did not fully understand the concept as evidenced in her explanation of the slope in the individual problem:

> OK, it's, it's going up, so, monthly return, well, the line is showing the correlation of the uhm, the plots, or the dots on the scatterplot. So it's showing that, I mean, there's a correlation, there's obviously a correlation, I'm not sure what it is, but it's probably, you know like. It seems like most of them are related. You have a few that don't really make any sense, but the, you know, it's not necessarily an indication that these are regular. But because all these are together, it's showing that everything, these are both directly related and they're both rising, so it's a positive correlation. So, I don't know if that's what you want to know (JessicaIPS, 114).

Even when questioned some more about what the slope meant, Jessica explained slope as "showing that they're all tending to go toward the same direction. So, it's not like jumping back down or jumping back up" (JessicaIPS, 120), a return to her correlation explanation. Jessica was not aware that she did not fully understand the concept of slope.

Statistics as a viable subject. Jessica realized she could use statistics in other areas of life besides fulfilling program requirements. She had organized a goal of doing what it takes to graduate and a sub-goal of learning statistics to help her understand other courses and other life experiences.

> I mean, I have to take a math for, you know. And I was thinking before I started the class, oh, I hate math, I don't want to do it, blah, blah, blah. But this actually relates to so much stuff. And especially with like polls and stuff. You never realize how inaccurate they actually are and stuff. . . . And now you're looking, and this applies to everyday life. So, yeah, I kind of see the purpose in it and it's like, I tend to be more motivated to do stuff if actually serves a greater purpose, I guess (JessicaIPS, 232-234).

Self-reporting. Because Jessica was a very talkative individual, she had no problem reporting what she was thinking, allowing her covert thoughts to be visible. She considered herself to be different from other students both in life experiences and in motivation to learn:

> Right, and most of them don't work. I mean, they don't, it's weird. [laugh] I feel really old sometimes and I'm not that old, but I'm like, I look at, you know, I'm in a different, I own my own house, I have a job, you know, stuff. So I'm in real life while I'm in college, but when I was in college before I was just like, you know, you don't have anything to fill your time except for you know, partying, and then if you do homework, that's a rare occasion. And I just think you don't care. I don't know. . . . But I notice in the general classes on this campus, people are sliding by, and they don't care. And it's, you know, it should, your major classes should be more concentrated, but people are underestimating the value of the other classes, too (JessicaIPS, 266, 276).

Jessica's attitude and talkative personality made her an open individual, willing to share with the researcher what she does to succeed. Her thought processes were visible in her conversation.

Metacognitive framework. Jessica's total evidences of metacognition, 53, was the second highest total for all seven students (see Table 4). She had a total of 15 for the class notes session, 24 for the individual problem solving interview, and 14 for the group problem solving interview. Jessica had just one noted lack of metacognition in the group problem solving when she said, "Yeah, well, I don't know what else you'd do" (GPSA, 230) as a response to a suggestion made by two other students.

In addition to the above-described metacognitive skills, Jessica revealed many instances that fit within Garofalo and Lester's framework. She was able to express strategies for analyzing and comprehending the given applications, "So I would know to know that for my test. I learn things through constant reinforcement, so I like write everything down continually" (JessicaCN, 6). Jessica came to class everyday so that she could practice and monitor her progress toward learning, "So, and it's you know, it's learning by, I can't learn math and stuff out of a book. I mean I really have to constantly have it pounded in my head. Literally, I mean and this is, this is practice, but this [the book] isn't where I learn it. I learn it in the class" (JessicaIPS, 206). She described the steps she took to solve problems and was a key participator in the group session. Jessica's method of verification was often asking other people in the group if they got the same answer as she did, but in the individual session she was able to explain why her answer was correct most of the time. Jessica had a total of 53 comments and/or behaviors (see Table 4) with comparatively large totals in both of her individual interviews, 15 and 24. Her group total of 14 instances is comparable with the totals of the other students who participated in her group. Her category for lack of metacognition has just one remark that was a response to which type of statistical test to use in the group problem solving interview: "Yeah, well, I don't know what else you'd do" (GPSA, 230).

Summary. Overall, Jessica used metacognitive strategies to learn probability and statistics. She could be considered an expert problem solver because she monitored progress toward her goals and posed critical questions to understand the situation at hand. She knew the best strategies for her to learn any subject were repetition and some memorization, resulting in excellent class attendance/participation and lengthy notes. Because she valued the potential of statistical knowledge, understanding was important to Jessica. This student made her thoughts visible by talking a lot, making an appearance that she used metacognition as a tool for her rather successful learning.

Amanda

Amanda scored 85.4%, the first quartile score, on the first midterm, and she earned an A- for the course. She was a very quiet person who preferred to work alone and knew this about herself. Her reluctance to share her thoughts resulted in only 16 pages of individual interview transcript (see Table 5), the least number of pages of transcripts in both groups of students; there were four pages of class notes and 12 pages of problem solving transcript.

Amanda was more of an observer than a participator in her group session and took notes in class only when she felt she understood something, resulting in very sparse notes (*i.e.*, one piece of scrap paper with six days of notes on three-fourths of the sheet). She attributed her classroom strategy to moods that she can not explain or describe: "Uhm, I don't know, its . . . some days I'm in a better mood to write notes than others" (AmandaCN, 30).

Novice vs. expert behavior. Amanda could be considered almost expert in her problem solving skills. She had very good mathematical and comprehension skills and posed critical questions about the process of solving the individual problem. Her understanding of the big picture, however, did not present itself; she never explained her answers in terms of the appropriate theories for the problem at hand. She did not verbally report that she checked to verify her answers were appropriate to the question posed, which is more characteristic of a novice problem solver. One example is when she was asked to explain the numerical answer calculated for the slope of the regression equation, Amanda replied, "Mmm, I mean I guess for every . . . I guess I'm not really sure what they're supposed to be saying, but" (AmandaIPS, 176). When the interviewer questioned her about a calculated answer, "OK, so you don't need to think about if it was . . . if it looks right, you feel all right about it?" Amanda simply said "Yeah" (AmandaIPS, 53-54). This response could have been due to her lack of interest or to her quiet nature or to both.

Statistics as a viable subject. The one idea Amanda did make clear to the researcher was her lack of interest in working at learning statistics. She took very sparse class notes and preferred to not interact with other students. Even though she was confident in her work she did not see a reason to learn statistics well enough to apply it in other situations. "Yeah, I guess I kinda feel fairly confident about the stuff I learn in class and I haven't been real interested in learning further. [nervous laughter]" (AmandaIPS, 206). This student had not yet recognized an opportunity to use statistical concepts outside the Elementary Probability and Statistics course.

Self-reporting. Amanda is a person of very few words. She demonstrated this by taking sparse notes in class, saying as little as possible in her individual interview, and observing rather than participating in most of the group interview. During the class notes interview Amanda was asked if she had her paper and pencil handy in case she wanted to take notes, and she replied that she did but "we were just going over the Excel thing and all the instructions are on Blackboard. I watched him do it, just so I'd be able to follow the directions myself later" (AmandaCN, 10). While she worked on the individual problem, it was difficult to get her to say out loud what she was thinking or doing. The following excerpt is typical of how the conversation flowed:

Interviewer	There you go. Take a minute to read it. You can use the book, the calculator, notes that you might have.
Amanda	OK [about 1 minute silence while S reads] OK [nervous laughter] This is sweet. Finding the least squares line. [pause] OK Oh! It's right there.
Interviewer	What's right there?
Amanda	a and b
Interviewer	How to find them, you mean?
Amanda	Yes.
Interviewer	Uh huh.
Amanda	All right.
Interviewer	So do you understand the problem?

Amanda	Mmhmm
Interviewer	OK
Amanda	OK, a is one point eight seven eight minus one point three o four times . . . oh … yeah so b is, is . . . a [nervous laughter] and b . . . is five one . . .
Interviewer	It looks to me like you're just plugging in numbers to the formula.
Amanda	Mmhmm.
Interviewer	From information given up at the top?
Amanda	Yeah.
Interviewer	Just from following your eye movements.
Amanda	Yeah.
Interviewer	Mmhmm, OK.
Amanda	[writing noise] OK, doing the b first.
Interviewer	And you think that calculator will get better answers or quicker answers than if you did it by hand?
Amanda	Yeah, I do in some places. Can I clear it?
Interviewer	Oh sure, yeah.
Amanda	All right. OK [calculator noise] Do you want me to talk it out while I'm doing it? (AmandaIPS, 7-32).

Amanda continued to work on the problem saying very little, even when questioned about what she was doing. And in the group session it was noted that "Amanda has not been doing very much besides watching the other three work" (GPSA, 174) and

"Amanda is just watching the others, waiting" (GPSA, 223). Other members of the group attempted to pull Amanda into the conversation with little success: "I always get those confused. Does that seem right to you? (to Amanda)" (GPSA, 213); and "So [pause] [pages turning] [Natalie, Cathy, and Jessica are reading their own notes. Amanda is just watching the others, waiting.] Seems like to me, well [pages turning] Does it sound right to you? (to Amanda)" (GPSA, 223). At one point Amanda did interject and convinced Jessica and Natalie to change a correct answer to an incorrect one.

Amanda	On the calculator it says mu does not equal forty and our H O is that mu does equal forty.
Jessica	Mm hmm, yeah, so we should change that I guess. Is that right? Yeah, so the H O would be not equal?
Amanda	I think so (GPSA, 210-212).

After further discussion by everyone in the group except Amanda, the group agreed to keep the original correct answer.

Metacognitive framework. Within the metacognitive framework, Amanda appears to have very few instances of evidence of metacognition (see Table 4). She has 15 occurrences in the individual problem solving which is more than twice her class notes total, 6, and her group problem solving, which totaled 5. The larger total could have occurred because the interviewer prompted Amanda to speak her thoughts. Compared to the other students Amanda's totals for class notes and the groups session are very low yet her total for the individual problem solving interview is similar to the other seven students. Her total for lack of metacognition was 10, a fairly high count compared to the other students' numbers in this category. Only one of her eleven instances in the orientation category was coded as comprehension of the problem; Amanda's understanding was addressing the given information and attempting to remember working with the same type of problem previously.

Her evidence of organization actually resulted in Amanda deciding she did not need to do anything. The interviewer asked, "But you didn't write anything down?" and Amanda replied, "Well, we were just going over the Excel thing and all the instructions are on Blackboard. I watched him do it, just so I'd be able to follow the directions myself later" (AmandaCN, 9-10). A few lines later Amanda commented "The reason I don't do it [take notes] there is because it's already on Blackboard" (AmandaCN, 24). She felt that the directions would be provided for her later, so she did not want to write notes that might be wrong. And her third coding of organization was the reason she wrote "analysis of variance" in her class notes: the instructor told the class "it would be on the test" (AmandaCN, 62).

Summary. Amanda provided evidence in the individual problem solving interview that she knew to think critically and to monitor her progress in order to be an expert problem solver but otherwise expressed an over-riding and decided lack of interest in learning/understanding beyond passing this course. She could recognize concepts and terms well enough to come up with acceptable solutions, categorizing her as a "successful" probability and statistics student as evidenced by her course grade of C or better. This person was a successful statistics student in that she could perform above average on examinations, however, her behavior and sparse comments gave the appearance that she did not try to understand reasons for following procedures. Amanda preferred to work entirely by herself and knew that was the most comfortable way for her to learn statistics: "I mean I haven't really tried much to study with other people. . . . But I tend to, I think I tend to look at things differently and I get frustrated when they're trying to focus on a different part of the question" (AmandaIPS, 222-224).

Natalie

Natalie's midterm score was 65.1%, very close to the third quartile of 62.6%. Her final course grade was B. Natalie was open to expressing thoughts that she could identify which resulted in 12 pages of transcript for her class notes interview and 12 pages for her individual problem solving interview (see Table 5), counts that are neither high nor low when compared to the other students' numbers. She expressed feelings of test anxiety and a lack of understanding why they happened. In addition, she admitted that she had not even reflected on why they occurred. When the researcher asked, "Are there times when you forget more easily or like is it big classroom, a small classroom, the teacher? Is there anything that affects that more?" Natalie's reply was "Uhm, I'm not sure, honestly. . . . No, I never really paid attention. I always just get really nervous before I take tests and then I'm like . . ." (NatalieCN, 73- 76).

This behavior is similar to her lack of making sense of any of her answers, computational or statistical: she only sought to verify her answer by asking the interviewer if she was right, not to make sense. Once Natalie finished one part of the individual problem, the researcher posed the question, "Does that make sense to you; is there any way you could check that by what you're given; or do you just assume that's right and go on?" Natalie's reply was "I just hope that's right" (NatalieIPS, 54). And in the group session, Natalie's participation was mainly comparing steps taken on a calculator. "What's underneath, see I don't remember how it is in the calculator" (GPSA, 110). At one point, she was even temporarily convinced to change a correct answer because she failed to make statistical sense of the numbers calculated.

Novice vs. expert behavior. Natalie was a novice problem solver. In addition to her not attempting to understand her anxious feelings when she took any type of test, she explained the reason for the order in which she answered questions, "I don't know I'm just doing it in order like on the paper" (NatalieIPS, 16). And a little later when she was asked why she wrote down the intermediate steps, Natalie said,

> I think it was just the way that I was trained. Like the way that I was taught because I know like when I was in high school math and other math classes, they always make you show like every single step (NatalieIPS, 40).

She was only aware that she answered the questions in the order they were encountered. There was no mention of posing her own questions or setting goals for solving the parts of the problem.

Often in class, Natalie was aware that she missed information presented by the instructor but did nothing to fill in the missing parts. She extended this difficulty to needing to practice to learn:

> That did help me. After doing it like a couple times I really like, I started to get it. And so, I got it on the test, like I remembered all of them from just doing it like over and over in class (NatalieCN, 134).

She was aware that the instructor provided many opportunities to practice problems last quarter but did not provide the guidance for the students this quarter. In Natalie's mind, this situation resulted in no other recourse, such as making sense, to successfully learn how to solve the problems:

> I mean last quarter he would uhm, put like practice problems on Blackboard and he would put uhm like the practice problems on the electronic reserves so then you could just print them out. And that made a huge difference when I was studying because then I could go through. And he would like later put the answers on there, like a couple of days later. So I could go through and practice like the ones that I knew and then, you know, go back and make sure I had those right on the, with the practice problems (NatalieCN, 136).

These novice characteristics extended to her problem solving attempts. In several examples, Natalie used no self checks of progress toward any implicit or explicit goal.

Interviewer	OK, uhm, do you stop and look? Does that make sense to you? Is there any way you could check that by what you're given? Or do you just assume that's right and go on?
Natalie	Well, honestly, like you could check it if you just like took two of the points off the, off the graph, but they're not all that easy to pinpoint.
Interviewer	Right, the raw data's not given to you.
Natalie	So, I just hope that's right.

Interviewer	Hope that it's right. Is there anything in there that would be a clue as to uhm how can I check to make sure this is right?
Natalie	I don't know. I don't know. If I remembered what all this stuff meant, maybe (NatalieIPS, 51-56).

Statistics as a viable subject. Natalie never verbalized occasions where she recognized statistics in her or others' experiences outside the classroom. She did talk at some length about how unfamiliar applications in statistical problems cause her to focus on all the information she does not know, causing a block in solving for the question posed: "Sometimes it's like, it's hard to like see your way past information you don't know. To like just the simple statistics of it" (NatalieIPS, 176). This statement is an indication that Natalie does not connect real world applications to the problem solving techniques taught in the statistics course.

Self-reporting. Natalie's language indicated she had trouble reporting her thoughts; her sentences were incomplete and she said the word "like" very often. Perhaps this was because her thoughts were confusing and difficult to pull together. Her inability to examine test anxiety (described above) and her confusing explanation of slope demonstrated that either she could not organize her thoughts or she could not put them into words to communicate them to other people.

> From like, over that period of time. Somewhat. It kind of, well like the line was positive and then when you look at the scatterplot and they kind of stay in like the same general area but it seems like the percentages are getting higher. If that makes any sense (NatalieIPS, 102).

Metacognitive framework. Table 4 reveals 38 instances when Natalie used metacognitive skills: 12 for class notes, 11 for individual problem solving and 15 for group problem solving. These counts are not noticeably out of the ordinary compared to the other six students' numbers, and her total for lack of metacognition, 10, is neither high nor low compared to the others' totals.

The additional category of "lack of metacognition" accumulated ten codings. For example, during the group problem solving session there were two occasions when the group was discussing the next step to take. After someone else made a suggestion for proceeding Natalie answered with "Yeah, that'd probably be a good idea" (GPSA, 167) and "Yeah, that's what I normally use" (GPSA, 235). There was no attempt to support or extend the suggestion made; she just agreed to continue in that direction. Overall, she used very few chances to make sense of the problem, to form a plan to solve, to monitor progress, or to verify any conclusions reached. Natalie simply pulled pieces from the information given to her, performed algorithms she remembered, and verified answers by asking someone else if "it was right."

Summary. Generally, Natalie used metacognition to perform arithmetical tasks which she identified as thought processes—"It [writing steps while problem solving] does help me actually because then I can go through like like if I get the wrong answer then I can go back and be like OK well this is what I got here. What did I do wrong? Like I can go back through my thought process to help me, so it helps" (NatalieIPS, 43)—but did not try to make sense of her answers. She only attempted to answer the questions posed using a recalled algorithm. Her self-reports were somewhat scattered which gives the appearance that Natalie's thoughts were also scattered. Whenever

Natalie paid attention to problem applications, it was because the situations posed unfamiliar information. She did not relate them to her world, which indicates she did not consider probability and statistics as viable for her life. Natalie earned an above-average grade in this course, because she followed algorithms. It appeared that she had not learned much about understanding statistical concepts; she only performed calculations that she learned in other situations to provide acceptable answers to statistically based questions.

Cathy

Cathy had the lowest midterm score of the group of students interviewed, 58.9%, somewhat below the third quartile of 62.6%. This student was also the only one of the group of seven who did not earn a better than average grade; she ended the course with a D-. Cathy's total number of pages for the two individual transcripts, 17, was the second least number of pages for all seven students (see Table 5). Her transcript had 8 pages of class notes and 9 pages of individual problem solving, which were both small counts compared to the other students.

Cathy was happy to take part in the study, but expressed hopelessness in her poor performance on the midterm exam. During the class notes review Cathy stated that she failed the first test "and I don't understand why. Because I got all A's in my psych stats and it's the same exact stuff" (CathyCN, 142). She did not understand that she must work past simply doing the steps involved in finding numerical answers. Cathy did not connect learning to interpreting calculated answers in a problem-solving situation. Metacognitive skills were present only for performing and recognizing algorithmic procedures she learned previously, and she did not understand she was not learning new statistical concepts.

During the individual problem solving interview Cathy calculated the slope of a least squares regression line; she was able to compute a number because she remembered the algorithm. Within the same part of the problem, she was not able to interpret the variation explained by the regression line, even though she did remember the correct algorithm.

Cathy	Yeah, I plugged in one point eight seven eight and then I subtracted the answer to these two [tap, tap, pointed to paper]
Interviewer	Oh, OK And so now you have an equation? A least squares line equation?
Cathy	mm hmm [pencil noise]
Interviewer	OK, great. And you took all this information just up here, from the given?
Cathy	Yeah
Interviewer	Yeah, OK
Cathy	[pause] I don't get it. [laugh]
Interviewer	OK
Cathy	What percent of the variation [pause] I don't get it.
Interviewer	The whole thing, you don't get.
Cathy	No, what are you asking for? The, what percentage? A percentage?

Interviewer	Yeah, that's good. So you understand percentages, right?
Cathy	Mm hmm, what percent of the variation, which is r squared. [pause] You don't just square r, do you?
Interviewer	Why not? [pause] What are you thinking about it?
Cathy	I really have no clue. What percent of the variation. I mean, I don't know, that would be my guess. That's what I would write. [never wrote anything for this part b]
Interviewer	What are you going to write?
Cathy	I'd write the answer to r squared, but that'd probably be wrong.
Interviewer	Why do you think that would be wrong?
CathyS	Because I feel like it's asking something else (CathyIPS, 68-86).

Novice vs. expert behavior. Cathy demonstrated novice problem solving behavior. As described in the previous example, she relied on feelings for determining the correctness of an answer. Individual questions were considered, but on her own Cathy did not relate the parts of the problem to the whole situation described. She was unconcerned about the meaning of the questions asked. When the interviewer asked if her difficulty in solving the problem came from a lack of understanding the stock market (the application involved) her reply was, "I wasn't really paying attention to the stock market" (CathyIPS, 114).

Cathy's external locus of control kept her from connecting a previously learned algorithm to a meaningful conclusion based on her numerical answer. She blamed her poor test performance on having a new teacher: "Yeah, cuz I never learned that before. I've always done other stuff, so he's new to me. He's a new teacher" (CathyCN, 90). Cathy did not even realize she was not learning statistics, because she felt successful learning in statistics depended on the teacher's assessments (*i.e.*, she had no control on improving her responses). "But I don't know why I'm doing so bad with him, but I got all A's in my other one. . . . And I don't understand why. Because I got all A's in my psych stats and it's the same exact stuff. I just don't like his tests" (CathyCN, 136-144). Cathy believed that no other mathematics teacher had ever required that she make sense of her calculated answers: "I've never had a teacher that just does that. I've just had a teacher that does that math. So, he's the first teacher . . . that's ever asked non-calculator parts. Like theory parts." The interviewer interpreted this comment to be "Ah, he's the first person to ask you to make sense of it" and Cathy answered with "Yeah, I guess" (CathyCN, 142-148).

Taking notes in class paralleled Cathy's lack of understanding; often she did not understand the ideas she was writing down and could not explain why she wrote them nor the notation that she used: "I don't know, that's how I write [laughing]" (CathyCN, 46) and "I don't know, I always do that" (CathyCN, 52). When asked how she would explain slope to someone new to statistics the following dialogue took place:

Cathy [pause] Like what it is or how to figure it out?

Interviewer Mm hmm

Cathy [pause] I could show them how to do it.

Interviewer OK

Cathy	I mean, I don’t know if I could really explain it. I don’t think I could explain it.
Interviewer	No? You don’t know it well enough to explain it to someone else?
Cathy	I mean probably. I just haven’t learned it in a long time, so.
Interviewer	OK, and so then that makes it hard for you to remember how to do these problems?
Cathy	I mean, like basic [tap tap] math skills, I can do, but
Interviewer	Uh huh
Cathy	Like I could tell people how to do equations and all that stuff, but like theory and stuff like, I don’t know. I’m not good at that (CathyIPS, 102-112).

Cathy had not yet developed a mathematical viewpoint with statistical understanding. Even when she described re-writing class notes as a strategy for learning, she said this does not happen in math.

> Yeah, I re-write all my notes. . . . Not not math, I don’t do my math that way. But like psychology and all that stuff, I re-write them. That’s how I learn. And when I write them, I get them better in my head (CathyCN, 116-124).

When prompted to continue with “you feel like it sinks in your head a little better?” Cathy stopped the conversation short with “[laugh] I don’t know” (CathyCN, 126) and returned to solving the given problem indicating she was uncomfortable explaining her thoughts further.

Statistics as a viable subject. The only instance in which Cathy was able to use statistical information in a somewhat novel setting was in her group problem solving session. In this example two students in the group had decided to use a null hypothesis of the population mean is equal to 40, which was a correct decision based on the given information. Amanda questioned the decision and convinced Jessica and Natalie to change the decision to the population mean is not equal to 40. Hearing this change, Cathy objected: "But for all of our examples we had H O is equal. Like for class and that" (GPSA, 217). After a few more seconds of checking notes and the text Cathy recalled "we always put it in the calculator, we always did not equal for that part [the alternative hypothesis]" (GPSA, 233). Even though Cathy was not necessarily making sense of the group's decision, she was able to make a connection to familiar information by recalling a previously learned algorithm to use with this application. She was able to communicate to the group that this was useful information for their decision-making hinting at recognizing statistics as a viable subject.

Self-reporting. As long as concepts or theories were avoided, Cathy was able to explain the problem solving steps she was taking. When the topic of conversation switched to statistical theory, Cathy was lost.

Interviewer	So these, why did you write this down again? It's the same thing as here. Why'd you do it again?
Cathy	This is a different example.
Interviewer	Uh huh, same stuff.
Cathy	No, it was different. This is different.
Interviewer	Different set of data?

Cathy	Yeah
Interviewer	But same theory that you were talking about?
Cathy	What do you mean?
Interviewer	Still talking about chi squared?
Cathy	Yeah
Interviewer	Mm hmm, and finding a decision. You still rejected null.
Cathy	Yeah
Interviewer	Mm hmm, so, it wasn't anything new, no new theory here, but you re-wrote another example down.
Cathy	Yeah
Interviewer	Why?
Cathy	Because he wrote another example. [laughing]
Interviewer	Another idea for you to look at?
Cathy	Yeah (CathyCN, 55-72).

When asked to explain the connection to statistical concepts, Cathy did not understand how to answer. Once the word "theory" was practically defined, Cathy used it other times to justify not understanding. "Like, I know how to do it. It's just, I don't know, I don't do good on theory, like the no calculator part" (CathyCN, 154) and "Like I could tell people how to do equations and all that stuff, but like theory and stuff like, I don't know. I'm not good at that" (CathyIPS, 112).

Metacognitive framework. There were 27 total codings of using metacognition (see Table 4) for Cathy. This number was the second to the least total of all seven students. Cathy's interview type totals were 7 for class notes, 4 for individual problem solving and 16 for group problem solving. In the last category, lack, Cathy had 16 occurrences, the greatest total compared to the other students for this category. For the class notes interview and the individual problem solving Cathy had a greater number for lack of metacognition than she did for her total metacognitions. This is the only student and the only interview type in which this occurred. (In the group session it was socially acceptable for someone to keep quiet when he or she could not contribute to the problem solving, but the student was obliged to admit when he or she could not use metacognitive skills in the class notes and individual problem solving.)

Summary. Overall, Cathy was a novice problem solver, relying on feelings and intuitions to supply some sort of answer to presented situations. She was able to self-report her thoughts and could even identify that which she did not understand. She explained her trouble was with theory, "the non-calculator parts," but was very comfortable with following algorithms. Cathy conceptually understood very little, including how statistics could be used outside the classroom. Based on her course grade, this student was the least successful at learning elementary probability and statistics of all seven students.

The students described above are the four students who comprised the first group problem solving session. The profiles that follow are of the three students who participated in the second group problem solving session, again in order of highest to lowest scores on the first midterm exam.

Maggie

Maggie had very little confidence in her mathematical skills even though she scored 88% (very near the first quartile, 85.4%) on the first midterm and earned an A- for the course. This student's algebra skills hindered her statistical learning mainly because she lacked confidence in her own ability. She did persist in trying to solve the problems resulting in nine pages from her class notes transcript and 14 pages from her individual problem solving transcript (see Table 5).

Because Maggie was so unsure of her mathematical skills she took many class notes (verbatim, if possible) and relied heavily on them to solve problems: "That's why I write down a lot. Like sometimes I'll even use his exact wording. Like, like there, that's how he worded it, and that's how I wrote it down" (MaggieCN, 156). She was explicit about her lack of confidence in her statement, "I'm not a math whiz, what? [laugh]" (GPSB, 423), when unsure of her correctly calculated answer.

Maggie's most notable strength was her ability to employ previously learned methods for understanding new problem situations. In an attempt to verify her answer to one question, Maggie input an equation into the calculator and compared her plot to the plot given in the problem, a strategy not discussed in class. When asked if she used many methods different from those used in class she responded, "No, but I kinda remember stuff from when I took statistics the first time . . . the first semester, at another school" (MaggieIPS, 101-103).

This type of behavior was repeated in the group session when she found answers using different methods than the other students. For example, when Maggie's group was confused about the steps to find standard deviation she was the only one who was able to plug the data into a list, find the answer, and use the answer to continue with problem solving: "cuz, I put list, like, the data and then is list one [showing calculator steps to Charlene]. And then the ninety percent confidence interval and calculate and that's what

I'm getting" (GPSB, 109). The other two students in this group were confused about how to proceed, but Maggie was persistent about her method: "See, but go to data. Cuz you have old statistics in there. I think, yeah. Now see what you get. See if I'm correct" (GPSB, 113). Maggie was successful in the class because many times she was able to use her class notes and previous learning experiences to recall appropriate algorithms and to make sense of calculated answers.

Novice vs. expert behavior. Maggie had characteristics of an expert problem solver. She recalled previously learned information to make sense of new problems and asked critical questions to work toward a definite goal. When asked if there was anything she could do to make sense of her calculated slope and y intercept, Maggie input the numbers in her calculator and recreated the same graph as given in the problem. This connected calculator skill allowed her "to make sure that my line is right" (MaggieIPS, 97). Maggie provided evidence that she regarded the structure of the problems posed: "Oh, this is the market, not, OK. So if the market is at, I don't really understand the stock market very well, but, so if it's . . ." (MaggieIPS, 207-209).

Oh the other hand, her lack of confidence kept Maggie from correctly interpreting some calculated numbers as they related to the problem, such as percents and slopes: "Uhm, I wrote down that it [correlation] could be between negative one and one. [pause] But I don't know how to figure a percent number, like uh" (MaggieIPS, 131) and

Maggie	[pause] what the slope of the line is, that is [pause] it's always the number in front of the x, right?
Interviewer	There you go.
Maggie	Yeah [pencil noise] OK

Interviewer	Does that number tell you anything?
Maggie	It's the rise over the run [laugh] so it's, it's positive so it's going up. And it's pretty
Interviewer	What's "it?"
Maggie	The line.
Interviewer	The line.
Maggie	Their, whatever we're, their stock. Is it testing whether or not it [pause] So, is it, if it responds well to changes in the market?
Interviewer	What do you mean by "well?"
Maggie	Like, it's, it says, the risk, but is it describing that it's a high risk? Or it doesn't have a high risk, because it's, it's steady through the market?
Interviewer	What does that slope tell you? You're not real sure?
Maggie	Well, I'm just not sure what, what the graph is.
Interviewer	Aah
Maggie	OK, OK, OK [pause] so your returns are generally pretty, it's pretty safe stock.
Interviewer	Why?
Maggie	Because, I mean there's like one real low outlier, but like if you were losing money, it would be, they'd all be in the negative, like, down here. Cuz you would make less returns (MaggieIPS, 187- 203).

Maggie's lack of confidence spilled into a dependence on her class notes and seemed to interfere with comprehending statistical concepts well enough to explain them accurately. She never did explain the slope of the line in this problem was the rate of change in the value of the individual stock as the overall market changed. Her struggle with explaining was a result of her lack of confidence in reading the graph. She not only relied on feelings for solving the problem, at times she allowed her feelings to interfere with sense making, resembling a novice problem solver.

Statistics as a viable subject. Maggie did not explicitly state that she believed statistics could be used in real life applications. She did imply that statistics was important enough to spend some effort learning it. Maggie's class notes were extensive. She wrote down everything the instructor said, verbatim if possible, and wrote given information in pen, calculated answers in pencil. These strategies helped her to decide what was important when she studied later. In addition, Maggie described a friend of hers who helped her with statistics: "But I have a friend who's a retired mathematician, so he worked for a law firm and did stats for them and stuff, so" (MaggieIPS, 153-157). This statement implies that Maggie realized the concepts being learned in class are used by other people, not just in a probability and statistics course.

Self-reporting. Maggie was fairly good at reporting her thought processes. She was able to explain all of her class notes and her thought processes in the individual problem solving interview. "Oh, I was thinking the line was over the b. OK [pause] [pencil noise] [calculator noise] [pencil noise] Can it be a negative number? Yeah. And that tells you which way the line goes, right?" (MaggieIPS, 56). Even though she had adequate math knowledge and excellent calculator skills, Maggie often expressed that math was difficult for her: "It's just that math, just cuz like, like he talks about intuitively you'll know, but I don't have math intuition. I just don't, like I have none. I can't do anything in my head" (MaggieCN, 162). She could communicate her lack of confidence and appropriate knowledge she recalled in support of calculated answers.

Metacognitive framework. Maggie's 52 metacognitive occurrences was a fairly high total (see Table 4), with 11 instances in her class notes transcript, 15 during the individual problem solving, and 26 in the group problem solving. Her totals for all but one of the categories in the framework are similar to the other students' category totals. Maggie excelled in Verification, and these instances were often from using reasonableness rather than retracing steps taken. She revealed only two times where she could have used metacognition but did not.

Summary. Overall, Maggie asked critical questions by making connections to previously learned concepts and algorithms. Her lack of confidence interfered with reporting her thoughts while she persisted in solving the given problems. Maggie realized that probability and statistics concepts were valuable knowledge outside the course. This student was successful in the course and in conceptual understanding of many probability and statistics concepts.

Charlene

Charlene's midterm score on the first midterm, 81%, was the lowest score considered at or near the first quartile (85.4%). She earned a final course grade of B. Her transcripts for class notes and individual problem solving were 5 and 16, respectively (see Table 5). The small number for class notes is the result of just one line of notes taken during the class on the day of the interview. Charlene only wrote "χ^2 on a 2×2 squared is the z test statistic" for the entire class period. She explained that she does not copy down the examples the instructor presents in class because "Like he'll just uhm put up a couple numbers and like find chi square and I don't see the point in writing that down when I could like look in the book for it" (CharleneCN, 29).

Charlene was straightforward when expressing that she sees no point in working at learning statistics, and she had little to say when explaining what she understood about concepts in statistics. She did not see value in learning beyond reproducing typical examples demonstrated in class, because she felt her choice in careers, athletic trainer, would not require understanding on the job. Passing the course was the only thing that was "important" to her this quarter. ". . . like, uhm, I got A's in my last two classes, uhm, so like now I guess I just don't, I'm not putting as much forth, as much effort as I should, or I was cuz . . ." (CharleneCN, 52).

Novice vs. expert behavior. Charlene showed signs of an expert problem solver by searching through the textbook for connections to the problem she was attempting to solve. It appeared she was working to understand the concepts involved in the situation at hand in both the group and individual problem solving interviews. During her group's session, Charlene spent a lot of time looking through the textbook, sometimes questioning what the other students were doing and why. Some of Charlene's statements found in the group session are: "Well, what are you guys doing?" (GPSB, 21); "I don't know what the standard error means" (GPSB, 57); "I don't understand the formula" (GPSB, 79); "I am stuck" (GPSB, 110); "I still don't know a thing that's on here" (GPSB, 114); and "Uhm, I really don't know what t or z means" (GPSB, 252). Charlene was attempting to develop meaning to be confident in the group's calculated answers, but she failed to ask critical questions of herself to lead her to a solution.

While solving the individual problem and after several prompts, Charlene attempted a brief explanation of her answer. When the interviewer commented that she did not seem confident in her answers, Charlene agreed but was not sure why she was not confident:

Interviewer But you don't seem real confident in that.

Charlene No.

Interviewer Why?

Charlene I really don't know (CharleneIPS, 227-230).

She did not express an understanding that slope is a measure of a relationship between two variables and, therefore, was unsure of her explanation of what the calculated slope meant: "But I don't understand what the slope of greater than one has to do with anything. Is that just like, would the slope be how much your stock is worth?" (CharleneIPS, 260). Charlene needed additional instruction in order to continue with her problem solving.

Eventually Charlene developed some meaning to the problems but not through her own actions. She knew the effort it would take to solve the problems appropriately but just had no interest, making it difficult to assess if Charlene was expert or novice in her problem solving attempts.

Statistics as a viable subject. Charlene was explicit about her opinion of statistics:

> Well, like, I don't think I'm ever going to need this. . . . Uh uh. I don't see like in, as a athletic trainer I'll use a regression line. . . . Well, yeah I think reports are important to like understand it, but I don't think like this in depth is that necessary (CharleneIPS, 138-146).

When asked why she did not give up trying to solve the individual problem, Charlene replied, "Well, that wouldn't help you any" (CharleneIPS, 328). She believed that the problem solving activity was important to the interviewer but saw no purpose for understanding statistics in her goals. She recognized the amount of effort it would have taken her to truly understand the work but was not willing to spend her time doing that: "Right, and yeah, this isn't a big priority" (CharleneCN, 54).

Self-reporting. Charlene thought she should answer the researcher's questions with "politically correct" answers. When questioned about her plans for studying, she timidly said,

> [embarrassed laughter] Yeah, I guess, cuz like for this test, uh, on Thursday, uhm, I probably won't open my book, because it's just going to be over chi square and that's mostly the calculator stuff, so . . . So I guess I'm kind of bad [at] that, I guess I should . . . (CharleneCN, 46-50).

Once she was reassured the interviewer was only interested in her thought processes, Charlene appeared to be more honest about her answers:

> I think it'll [note-taking in class] be the same. I'm only going to write down like stuff that I think I'm really going to need for the test. Like, uhm, for chi square, how you find it. Just observed minus expected, like that (CharleneCN, 60).

During the individual problem solving session, Charlene explained steps she took on the calculator as she did them and identified steps she could not remember how to do on the calculator. She could explain when she might use a calculator to check for mistakes in her calculations: "If I'm like really not sure about it or if I thought it was like too easy, I go back and double check" (CharleneIPS, 106). In contrast, she spent much of the time in her group session searching through the text and only communicated her thoughts when she felt confused or when she felt she might have had additional information to offer. "What is the critical value, confidence interval, the confidence interval, don't you go to uhm, test and a z test, number one?" (GPSB, 36) was one of her few contributions and was about following a procedure to calculate a numerical answer.

Metacognitive framework. Charlene's totals of 5, 17, and 30 for class notes, individual problem solving, and group problem solving evidences of metacognition appeared to be relatively high compared to the other students' totals (see Table 4). However, of her 52 metacognitions, 14 were from analysis of information in the group problem solving session which had a total of 30. During the second group's problem solving session, Charlene often expressed she did not understand what was going on and spent a lot of her time looking through her book for information to help her. It was noted 13 times that Charlene was looking at a textbook (G2GPSB, 72; 82; 93; 139; 144; 152; 186; 204; 215; 225; 374; 387; 392). During data analysis it appeared Charlene was looking for meaning about the problem but, considering her explicit lack of interest, perhaps she was just trying to appear involved during the interview. If the 13 comments about not understanding were moved to the category Lack, Charlene would have had a total of 39 metacognitions and 15 for Lack.

Summary. Charlene had made a decision that she did not need to learn statistics for her career goals which may have kept her from being an expert problem solver in this study. Because she was not confident in the answers she did provide, Charlene persisted in trying to appear as if she were searching for solutions. When her group session was over, Charlene had no interest in hearing how they did as a group; she only wanted to leave. She made it clear that she was willing to help with the study but had no personal interest in understanding statistical concepts. This depicts very superficial use of metacognition, not a tool for successfully understanding the concepts.

Mark

On the first midterm Mark scored near the third quartile (62.6%) with a 60.4% and finished the quarter with an A. His total number of transcript pages for the class notes and individual problem solving interviews was 20, ten pages for each interview (see Table 5). Mark blamed his poor performance on the midterm to paying more regard to the book rather than the instructor's lecture. He explained that in previous classroom situations, knowing statistics was "just a matter of knowing how to multiply, you know, and add." (MarkCN, 112). At the beginning of the quarter he was unsure as to how to learn from his instructor's style of teaching (applying concepts), but once he realized that statistics applied to other things he wanted to learn, he performed very well on the assessments. During the class notes interview, Mark explained his plan for staying focused in class:

> Well . . . I mean I sit try to sit towards the middle or the front of class so I don't see what everybody else is doing, so . . . Well, if I sit in the back I

can't hear, so that's part of the problem. Sitting in the front just helps me better. I can see better, hear better (MarkCN, 22-28).

After taking this statistics course, Mark related that he understood more statistical concepts and had examined what it takes to be successful. He expressed his concern for understanding calculated answers both in the individual and the group problem solving sessions.

At times Mark appeared to mimic others' words rather than think for himself, but perhaps this is one of his strategies for understanding the concepts.

Interviewer I think that the main idea of using a calculator is to get away from the grind of subtract this, square this, divide by this. What do you think about it? Does it help you concentrate on the more important concepts?

Mark It gets you away from the grind of actually doing all the arithmetic that's involved, and lets you focus more on the content that's going on (MarkCN, 129-130).

Novice vs. expert behavior. Even though Mark did not score well on the first midterm, he could be considered near expert in his problem solving skills. By looking for unmentioned details in the instructor's lecture, Mark paid attention to the structure of information presented:

Interviewer All right, 'chapter two info,' why did you write that down?

Mark No, I just thought we, cuz I was trying to find out where he was getting like the correlation and regression stuff from. And I noticed on the slide that was on the screen for the

	scatter plots that it was chapter two. So I just jotted that down so if I have any questions about it at all, I can just go back to chapter two and look through it.
Interviewer	Mmm, good thinking. So you didn't get your book out and look. You just saw on his notes.
Mark	No, I just saw on his notes.
Interviewer	Do you look around on his notes for extra stuff that he might not point out? Is that something you do?
Mark	I always do that (MarkCN, 37-42).

When Mark recalled previously learned information about new material he wrote it down to "jog his memory" (MarkCN, 86) and to build on the schema he had already developed.

It was important to Mark to understand the problem situation and to make sure calculated answers seemed reasonable to the application.

> I'm a little concerned. I'm trying to figure out. I can't remember, what's the difference between a z and a t? [*to Maggie*] Doesn't a z have a, don't you know the standard deviation? You're looking for. You don't happen to know, you're looking for standard deviation (GPSB, 87).

When Mark's group was struggling to find the confidence interval he was able to calculate and verify the accuracy of his calculation of a range:

> So, the critical region would be thirty-five point eight, plus or minus two point nine seven. Which I think comes up to thirty-two point nine five o [calculator noise] thirty-eight point seven seven to thirty-seven point eight, or thirty-eight point eight and [calculator noise] thirty-two point

> eight three. . . . That's how you get your confidence interval is x bar plus or minus z star. . . . Yeah, that's pretty close. Calculated with rounding error. But that's all you do is, is I mean it's x bar plus or minus your standard error. And then you get your critical or your confidence interval. It's coming back to me now, sort of (GPSB, 168-177).

Statistics as a viable subject. Mark recognized experiences outside the statistics classroom where he could use the concepts presented.

> Well, I mean, there's, I don't know, I'm kinda, I like statistics. There's many reasons to eventually use statistics in history. You know, and ah, I know there's like some political science classes that I've taken. Uhm, and they have a lot of statistical stuff in there. But I didn't understand. Now I'm starting to understand, yeah, how they got those numbers, and why they got those numbers, and what they actually mean. But before I didn't have a clue what they meant, so (MarkCN, 140-142).

Not only did he find concepts viable, he valued understanding how numbers were calculated and their meaning. He even purchased the recommended (expensive) statistical calculator when he already had an acceptable one.

> Right, yeah, and it's just, it's easier doing it by the calculator. It gets you away from the grind of actually doing all the arithmetic that's involved, and lets you focus more on the content that's going on . . . instead of worrying about, you know, adding and dividing, you know, all that stuff. You can focus on the content and know what's going on, you know, just as well as doing all the arithmetic, so (MarkCN, 130-134).

Self-reporting. Mark appeared to be able to verbalize his thoughts clearly. He had no trouble explaining slope in the stock market application even though he is not interested in stocks:

> Uhm, I'm not really interested in stocks, no, to be honest with you. I mean, I understand it. . . . It took me a while, I had to really think about it. . . . Yeah, it was coming slowly, so, I mean, it was getting there (MarkIPS, 161-170).

Mark did not let his lack of interest in this application hold him back on making sense of the graph of the stock compared with the overall average. He used his metacognitive skills to perform calculations, made sense of the answers, and communicated what he was thinking.

Mark's self-reporting skill was more apparent in the structured one-on-one situations than in a group. At times, the students in Mark's group talked in parallel. Mark, especially, did not appear to be communicating with the others, just with his own thoughts.

Maggie	This, did you do this?
Mark	Well, that's the standard deviation.
Maggie	Yeah, oh, right, right, right.
Mark	I didn't put it in my list. I just did it manually. I'm doing it now.
Charlene	What is the critical value, confidence interval, the confidence interval, don't you go to uhm, test and a z test, number one?

Mark	Mm hmm
Maggie	No, that's not right. [Opening textbook]
Mark	t interval
Charlene	Mm ninety [pause] OK for d, I got negative twelve, err, negative twelve point nine seven and eighty-four point
Maggie	Where'd you go? Did you go to z interval?
Charlene	Yeah
Mark	What'd you, ninety percent?
Charlene	Uh huh
Mark	Here you go ??? [inaudible]
Charlene	If you got to change anything it's up to the c level.
Mark	Yeah
Maggie	Hmm, mine didn't, put [pause] Here we go.
Mark	Let it do it all? (GPSB, 32-51).

Metacognitive framework. Mark's total number of metacognitions was 59 (see Table 4), the highest total of all seven students. His class notes total was 7, and the individual problem solving transcript reveals 17 occurrences of metacognitive behavior. Thirty-five of the 59 were in the group session; 13 of those 35 were indications that revealed looking at the information to comprehend the situation, and 15 of the 35 were verifying calculations were done correctly. There were only five instances where Mark could have used metacognition but did not. One of those, from the individual interview,

> Is explained by linear regression equation of y, with the market as a whole, x. I don't know, I just, that question's throwing me. I don't know

> why. . . . I don't know if it's the way it's worded or what, but I don't understand what it's trying to get at (MarkIPS, 64-66).

resulted in a discussion between the interviewer and Mark. Eventually Mark did understand the question but at that particular point he gave up without referring to the text or asking for help.

Summary. Mark was aware of his best learning strategies and utilized them to understand concepts presented in the class. He posed critical questions while attempting to solve problems and used understanding rather than repetition of steps to verify his answers. Mark's ability to self-report his thoughts appeared to be veridical across interview types. After taking this class and working at understanding, Mark knew statistics was a viable subject for goals outside the class.

Student Categories

After the data for the seven students who participated in the out-of-class interviews were analyzed, it was apparent that the students could be grouped in a way different from that described above. Two categories evolved. The first category included Mark, Jessica, and Maggie, because they demonstrated characteristics of expert problem solvers, recognized benefits of knowing statistics outside the classroom, were fairly competent in reporting thoughts, and provided the most evidence of using metacognition while learning elementary probability and statistics. The second category of students—Amanda, Charlene, Natalie, and Cathy—demonstrated novice problem solving characteristics, either denounced statistical knowledge or failed to observe its existence outside the classroom, lacked self-reporting skills and/or abilities, and provided less evidence of metacognition use than the other category of students.

In the first category, Mark gave the only evidence of using metacognition to conceptually understand how situations related to the statistical concepts that were present in the situation at hand. Jessica and Maggie were not as successful in their conceptual understanding but did make sense of the problems as they interpreted them although not always in a completely accurate way. The students in the second category were less successful at conceptually reporting elementary probability and statistics concepts as those in the first group, either from a lack of considering learning beyond computations or from a lack of wanting to understand concepts further.

Relative frequency of each student's contribution in the group problem solving transcript is presented by the new categories in Table 6, for comparison purposes. The

Table 6.

Student information, interview contributions, and framework counts

	Midterm Quartile	Group P.S. Session	Group Session (% of Total)	Class Notes (pages)	Individual P.S. (pages)	Total Meta-cognitions	Total Lack	Course Grade
Category 1								
Mark	3	B	34.7	10	10	59	5	A
Jessica	1	A	42.4	13	15	53	1	B+
Maggie	1	B	43.5	9	14	52	2	A-
Category 2								
Amanda	1	A	6.1	4	12	26	11	A-
Charlene	1	B	21.8	5	16	52	2	B
Natalie	3	A	30.6	12	12	38	10	B
Cathy	3	A	21.0	8	9	27	16	D-

table also presents data from Tables 4 and 5 (framework category totals and the two individual interviews page counts) and each student's final course grade. The following discussion summarizes both categories of students according to the four themes: novice vs. expert behavior, statistics as a viable subject, self-reporting, and metacognitive framework.

Novice vs. expert behavior. Mark, Jessica, and Maggie, showed signs of being expert or nearly-expert problem solvers. They focused on the structure of the problems and asked questions about the process of solving the problems presented. Attempts at explaining the meaning of calculated answers were made by all three students in the first category.

In the second category of students, Amanda and Charlene both stated they did not have an interest in probability and statistics and therefore only attempted to answer questions by performing algorithms that provided acceptable answers. Natalie and Cathy provided evidence that they were unaware of why they could not answer questions; if they met a dead end while problem solving they might retrace their steps but did not consider alternative methods that might work. Metacognitive skills were not developed enough to be considered a tool for success.

Statistics as a viable subject. The students in category one all explicitly stated in some form that statistics applied in situations away from the academic course. Mark personally experienced how statistics fit in with his studies in history. Jessica explained that specific information about polls and other life experiences helped people to understand the situations: "this applies to everyday life" (JessicaIPS, 234). Maggie described a friend who used statistics in his career in a law firm. In addition, these students all implied an interest in understanding and used available resources to learn probability and statistics concepts.

Two of the students in the second group saw no point in learning statistics beyond succeeding in the course. Amanda made it clear she had no interest in working any harder than she did in order to understand the concepts, and Charlene felt in-depth understanding would not be helpful to her as an athletic trainer. The other two students appeared to be unaware of the benefits of learning probability and statistics. Natalie associated the feelings she had in this class with other classes; she was anxious during tests which made her perform poorly. She tended to focus on not understanding even the non-statistical information, revealing she did not connect these concepts with life experiences. Cathy could not even connect her performance in this statistics course with another statistics course she took in the past. She seemed to be the student who considered her own cognitive thoughts the least of the seven in the study. These four students either were not convinced statistics could help them with their own personal and academic goals or were unaware of how statistics could be useful outside the classroom.

Self-reporting. The number of pages for each students' transcripts by type of interview provides a rough estimate for comparing how much each student was willing to share his or her cognitive thoughts (see Table 6). In the first type of interview, class notes, Natalie was the only student in the second category who had more pages (12) than students in the first category (totals of 10, 13, and 9). Otherwise, students identified in the two new categories had similar counts. The number of pages for the individual interviews are also similar within the categories except that Charlene (category 2) had the most pages of all the students, and Mark (category 1) had fewer pages than everyone except Cathy (category 2).

The third column's numbers were calculated by counting the number of lines identified for each student within the conversation during his or her group problem solving session divided by the total number of occurrences of all the students' comments in the same group. For example, problem solving Group A (Amanda, Cathy, Jessica, and Natalie) had a total of 229 lines of transcript, and Jessica's lines of comments during the session numbered 96, resulting in a 42.4% contribution rate. These numbers were also calculated as a rough estimate of how much each student was willing to verbally participate within a group problem solving session. The first category of students, Mark, Jessica, and Maggie, all participated at a higher rate than anyone in the second category.

The students in category one—Mark, Jessica, and Maggie—were all able to express their thoughts fairly clearly. Mark had very little trouble explaining what he was thinking and what the answers meant in terms of the application at hand. Jessica and Maggie were both very clear about classroom behavior and thoughts that moved the problem solving process forward.

The students in category two—Amanda, Charlene, Natalie, and Cathy—required some speculation as to reporting skills. Amanda did not report many of her thoughts at all. It was not clear if this was due to her personality or to her lack of interest in studying probability and statistics. Charlene verbalized her thoughts when she thought it would help with the researcher's goals. She spent a lot of the interview sessions looking through the textbook or expressing a lack of understanding. It took a lot of prompting to get her to speak about her thoughts. Natalie attempted to say what she was thinking, but her reports were unorganized and lacked understanding beyond algorithmic procedures. The fourth person in this category, Cathy, reported a lack of awareness of her learning strategies and applicability of the problems, she did not understand why she was not doing well in this class.

Metacognitive framework. The tally of all the counts of metacognition are also presented in Table 6 by student category. Once again the counts of the number of times metacognition appeared in the data sorts the students into the same two categories, except for Charlene. A possible reason that her totals appear to be an anomaly is given below.

In the first category, all three students—Mark, Jessica, and Maggie—had at least 52 total pieces of evidence of metacognition and fewer than six comments indicating metacognition could have been used but were not. Three of the students—Amanda, Natalie, and Cathy—in the second category had far lower totals for metacognition—26, 38, and 27—and their indications of a lack of metacognition were higher than the first category—11, 10, and 16. Although Charlene's overall comments and behavior categorize her in the second group, she had a total of 52 notations of metacognition and two for a lack of metacognition, similar to the first category of students. A closer look at the group problem solving revealed that 13 of the comments could have been coded as lack of metacognition, depending on interpretation. If these 13 comments were re-categorized as "Lack," Charlene would have an overall total of 39 metacognitions and 15 opportunities that did not utilize metacognition but could have, more in line with the second category of students.

Summary

The seven students interviewed outside the class provided varying evidence of metacognition while learning elementary probability and statistics concepts and achieved various levels of success in the course, as defined in the introduction of this study—earning a C or better. Comparing the student categories by course grade might reveal that metacognition is necessary only for some students. However, comparing the students by interview contributions and the four emergent themes—novice vs. expert problem behavior, statistics as a viable subject, self-reporting, and the metacognitive framework—points to a different definition of success, conceptual understanding.

Chapter Five

Discussion

Research about probability and statistics students reveals many common misconceptions and mistakes in interpreting the concepts presented (Batanero, Godino, Vallecillos, Green & Homes, 1994; Fischbein, 1975; Kahneman & Tversky, 1982; Tversky & Kahneman, 1971; Well, Pollatsek, & Boyce, 1990). Research about mathematical problem solving has generated some ideas about what the students are doing and need to do when learning to solve mathematical problems (Campione, Brown, & Connell, 1989; Garofalo & Lester, 1985; Lester, 1989; Schoenfeld, 1985, 1987; Shaughnessy, 1985). Paying attention to the application of the problem, organizing a plan for answering the question posed, and verifying that the calculated answer is appropriate (the metacognitive aspects) often are missing in problem solving instruction for various reasons but may be useful for successful learning (Schoenfeld, 1992; Shaughnessy, 1992). According to the constructivist learning theory, students organize new information by building on knowledge already assimilated, but for each student the process is different (von Glassersfeld, 1996). Therefore, some students may need to pay attention to the metacognitive processes they are using to learn successfully, including probability and statistics theories and applications.

Before this study was conducted it was believed that students who employ metacognitive skills to learn probability and statistics are more successful at learning concepts presented in the course. A successful student was defined as one who receives grades of C or better. The results of this study found students used various levels of metacognition while learning probability and statistics concepts. The level of success, however, was not consistent with using, or not using metacognition. The seven students interviewed provided evidence of using different amounts and kinds of metacognition but earned similar course grades: 3 As, 3 Bs, and 1 D. Supporting the hypothesis somewhat, the seventh student, who received a D- for the course, was the one student who used metacognition the least.

Theoretically, constructivism states that everyone is different in assimilating knowledge necessary for successful learning, which was evident in this study. Mark, Jessica, and Maggie provided much evidence of metacognitive thinking and earned grades of A, B+, and A-, respectively. Amanda and Charlene provided less data that were coded as one of the framework categories; they earned A- and B for course grades. The two students who provided very little evidence of metacognition, Natalie and Cathy, earned B and D-. The different levels of metacognition used did not relate to performance on assessments. The relationship that did become apparent was using metacognition and reporting the new stochastic knowledge assimilated into each student's schema of constructed knowledge. Each student's level of reporting conceptual understanding coincided positively with metacognition evidence.

Students' Metacognitive Skill Use

In order to investigate the role of metacognition in learning stochastics concepts, three guiding questions were established at the beginning of this study:

1. What are the students doing cognitively to learn concepts in elementary probability and statistics?

2. What role does metacognition play when students are learning how to make decisions that require an understanding of probability and statistics concepts? and
3. Is metacognition a necessary element for success in undergraduate elementary probability and statistics courses?

This chapter interprets the data to answer the three questions. How this information is useful is then presented by the four themes found in the literature, followed by a brief description of the limitations of this study. The chapter concludes with thoughts on future research that might reveal additional information about the role of metacognition while learning elementary probability and statistics concepts.

What are the students doing cognitively to learn concepts in elementary probability and statistics?

The students brought to the classroom various backgrounds in mathematical and reading comprehension skills developed through previous educational experience. In addition, they each cognitively made a decision that Elementary Probability and Statistics was a course they needed for graduation in their respective programs. After that decision was made, most of the students consciously made a decision, *a priori*, regarding the usefulness of learning the concepts presented. Jessica, Mark, and Maggie all gave examples of how probability and statistics concepts were useful in areas outside the classroom and stated that this course information would be useful to them in the future. Amanda and Charlene explicitly stated that probability and statistics was not a viable subject as far as they envisioned their future. Natalie and Cathy were the only two students of the seven interviewed outside the classroom who did not provide evidence they were aware that they could make this decision. Throughout the course, these seven students used previously constructed knowledge in mathematics and reading and their belief about the usefulness of the course in order to complete assessments in an effort to be successful in the course.

In the classroom students, including those who were interviewed outside the classroom, appeared to be aware of acceptable classroom behavior. They either paid attention to the instructor and participated in class discussion or gave the impression that they were listening. Many students found other activities to do while instruction took place, such as read other subjects, play games, chat quietly, or sleep but they did not disrupt the other students' learning. The students who were observed participating in the classroom setting watched and listened to the instructor as he explained and wrote solutions to sample problems in front of the classroom. At times the students answered the instructor's questions and also posed some of their own to the instructor and/or to each other. The students who were interviewed outside the class regularly attended and paid attention.

In the interviews the students explained their class notes, read the problems, and made connections to previously constructed knowledge. Cognitively they heard and/or recalled information that resulted in notes taken and algorithms followed. All seven students cooperatively communicated with the interviewer by answering questions and attempting to solve the problems posed.

Overall, the students brought previously constructed mathematical and social behavior knowledge to the classroom. In addition, each student's beliefs about the course influenced the amount of work and participation he or she felt was necessary for meeting personal and academic goals.

What role does metacognition play when students are learning how to make decisions that require an understanding of probability and statistics concepts?

The students who participated in class provided some evidence of metacognition, but because thinking is a covert activity, qualitative methods were necessary for uncovering thoughts and identifying learning. The seven students who participated in the out-of-class interviews provided the bulk of the analyzed data. These seven students used metacognitive skills at various levels. Mark paid attention to his learning progress the most. Maggie and Jessica were fairly similar in their levels of metacognitive skill use, providing evidence less successfully than Mark but more than Amanda and Charlene. They were able to make sense of questions asked and found ways to verify computed answers. Amanda and Charlene's evidence of metacognition was highly constrained by their explicit belief they were not interested in understanding the concepts. Natalie used metacognitive skills very few times and in limited ways, and Cathy provided the weakest evidence of metacognition compared to the other seven students. Garofalo and Lester's (1985) categories of metacognition provided codings for the evidence gathered in and analyzed across the three types of interview sessions.

Orientation. Comprehending the situations at hand, analyzing the given information and conditions, recognizing similarities to previously worked problems, and representing the facts are all subcategories for the first category in the framework. Few of the students even paid attention to the context of the problem being solved. Comments like "I wasn't really paying attention to the stock market" (CathyIPS, 114) and ". . . It doesn't say which one to use" (G1PS, 123) were some of the indications that comprehension of the problem at hand never took place.

Two students provided explicit evidence that understanding the situation at hand took place before working at answering the questions: "I was just trying to think, I just, I was just trying to figure out what in the world the question's really trying to ask" (MarkIPS, 74) and "Yeah, like I just, like it's better with me if we, if I learn, like through examples. . . . Like when he puts stuff on the board and just has like ah like a chart up there, and all the numbers are already filled in and stuff, I'm totally lost. Like, you might as well, I may as well not even listen. Or like reading that when it's generic. Like a lot of x and y's, you know, like, that means nothing. I need to, like, see what it, what it's doing" (MaggieIPS, 161-163).

Organization. This category involves plans for solving a problem including setting goals or sub-goals during the process. At times the students in this study made a plan for answering the question posed, but that was usually considering which equation or tools (class notes, textbook, calculator) to use. In the first group problem solving session, group one decided to use a t distribution equation rather than a z distribution only because the problem asked for standard error, a number associated with the t distribution; no mention was made that a z distribution was inappropriate because the population standard deviation was unknown (G1PS, 62-63). Maggie's lack of self-confidence resulted in strong reliance on her class notes for appropriate problem solving. Charlene referred to the textbook throughout both problem solving sessions which at first appeared to be metacognition. Through data analysis, it was revealed that this behavior may have been Charlene's attempt to appear to be problem solving; she had not previously constructed enough information to answer the questions. Natalie performed algorithms in the order that they were required in the presentation of the problem. She and the other students provided very little evidence of making goals or sub-goals before jumping in to answer a question.

Execution. Performance of local actions or monitoring of progress of local and global plans is required in execution. The students recalled algorithms relevant to the information presented in the problems and calculated their solutions. Because few goals were identified in the organization category, monitoring of progress toward a goal was not observed. Most of the evidence of the students' monitoring progress of mathematical computations was metacognitive skill learned prior to attending Elementary Probability and Statistics. The only time a student revealed considering trade-off decisions was Mark's comment on the expediency of problem solving: "I just wanted to see what the problem was first before I got started. . . . Uhm, see what would be the quickest way to answer it" (MarkIPS, 8-10).

Verification. The students verified their algorithmic procedures mostly by comparing final numerical answers with someone else's answers, not by reflecting on the statistical concepts presented in the course. For example, some of their comments were "What'd you put?" (GPSB, 412) "What'd you get," (GPSA, 183) "So t is negative two point one four? Is that what you got," (GPSA, 184) and "Yeah, that's what I got" (GPSB, 28) During the individual problem solving Natalie made a typical comment with, "So then I got, so the formula would be y equals one point one seven plus point three five two x. It'd be like the least squares line. The equation. Is that right?"

(NatalieIPS, 50). Once again with no reflection on the calculated answer, the student sought confirmation rather than comprehension.

One student had progressed to conceptually explaining numerical answers computed with statistical algorithms. Mark was able to report meaning about a stock market question by defining slope accurately; he made sense of what a number told him about an application with which he was unfamiliar. Both Jessica and Maggie made connections to other previously constructed knowledge to explain why their calculations were correct: "Yeah, well, I'm, when I see something like that I automatically want to solve for every letter. . . . So, I'm thinking I don't have to do that because I'm finding the equation, not the answer" (JessicaIPS, 72-74) and "Oh, oh, oh, oh. I was thinking the line was over the b. OK [pause] [pencil noise] [calculator noise] [pencil noise] Can it be a negative number? Yeah. And that tells you which way the line goes, right?" (MaggieIPS, 56). In contrast, Natalie commented that she wrote down the steps in a problem because she was trained that way: "I think it was just the way that I was trained. Like the way that I was taught because I know like when I was in high school math and other math classes, they always make you show like every single step" (NatalieIPS, 40). This student had trouble conceptually explaining or making connections to anything during her interviews.

Overall, each student's personal decision as to whether or not probability and statistics is a viable subject influenced how much and what types of metacognitions were employed to learn the concepts. The students who recognized use of stochastics outside the classroom—Mark, Jessica, and Maggie—provided more evidence of metacognitive thinking than the students who either believed stochastics was not important—Amanda and Charlene—or did not consider real world applications—Natalie and Cathy.

Is metacognition a necessary element for success in undergraduate elementary probability and statistics courses?

The plan was to compare high achieving students to low achieving students, but all of the interviewed students except Cathy received As and Bs for course grades. Six of the seven students (85.7%) may be considered successful as defined by this study, because they earned a grade of C or better. Either this course grade only reflected recollection of appropriate algorithms or some sort of intervention took place for the students agreeing to participate outside the classroom. The former position is supported by the whole-class grade distribution of 80.9% successful students. And comparing the students who performed near the first quartile to the students who performed near the third quartile on the first midterm exam did not work either. Two of the three students near the lower quartile earned an A and a B for the class. The student who scored the lowest on the first midterm was the only student participating outside the classroom who earned a D- for the course grade. Although the students used metacognition in various degrees and ways, and most of them earned successful grades in the course, it became apparent that many theories presented in the classroom were not assimilated into the students' schemas of mathematical problem solving.

Only by qualitative analysis, listening to the students self-report their thought processes, was conceptual understanding revealed or not revealed. The behavior that was observed for each student was common across the three types of interviews. For example, Mark explained thought processes in the classroom in his class notes interview; he paid attention to the best strategies for him to learn. In the individual interview, Mark correctly and succinctly explained the information he gleaned from a computed number; he went beyond algorithms to make sense of the answer. And in the group problem solving session Mark explained to the group why his confidence interval was correct by relating it back to the question asked, not by simply repeating the steps he took the first time.

On the other hand, Cathy's behavior puts her at the opposite end of using metacognition to learn statistics. She was completely lost as to why her assessments received such low scores. Not only did she not examine her thoughts, she found external sources for causing her to perform badly; *i.e.*, it was the instructor being new to her that caused her to not do well. She even admitted that she could only show other people how to calculate answers to questions, she could not explain how to do this and she could not explain why it was important. During the group session, Cathy did convince the other students to return to a correct answer that she recalled from the classroom. She made the connection that in class the null hypothesis was always set to zero, which could have been a lead into her plan to solve the problem.

Overall, each of the interviewed students provided some evidence of metacognitive thinking and six of these seven students earned a C or better for their course grade (this study's definition of successful). Further classification of students' use of metacognition and success reveals that the three students who provided the most metacognitive data were successful and only three of the four students with less evidence were successful. Qualitative analysis revealed the students with more evidence of metacognition could conceptually explain their thinking better than the four students with less evidence. Depending on the requirements for success in a course, it appears that using metacognition is not necessary for success; it may only be helpful for some students.

Importance of the Study

This study addressed Shaugnessy's (1992) question about the role of metacognition in learning probability and statistics concepts. For these seven students increased metacognitive thinking appeared to improve conceptual understanding or, perhaps, self-reporting of conceptual understanding. The relationship between using metacognition and succeeding in the course was not evident for the students in this study. They did, however, provide evidence in each of the four themes from the literature that warrant discussion.

Novice vs. expert behavior. An expert problem solver pays attention to the structure of the problem and asks critical questions in the process of answering the questions posed. Both of these criteria require metacognitive thoughts. If educators expect students to develop into expert problem solvers, explicit direction in paying attention to making sense of progress made and improving self-monitoring during problem solving should be addressed. This study showed that the interviewed students who used metacognitive processes more often had characteristics of expert problem solvers.

Statistics as a viable subject. Students who have made the decision that probability and statistics concepts are valuable outside the classroom have a reason for understanding applications and checking reasonableness of calculated responses to questions. The three students in this study who identified where they could use the concepts elsewhere were the three who employed metacognitive thinking more often.

Self-reporting. Even though self-reporting is full of scientific controversies, it is the only process available for uncovering students' covert thoughts. Through data analysis of the students' interview transcripts, metacognitive thoughts were revealed only if the students reported them. Each student's self-reporting efficiency was consistent across the interview types. Students who revealed their thoughts in the individual interviews tended to contribute more to the discussions in the group interviews. Conclusions about the students who did not or could not report their thoughts required some speculation based on other behavior, such as identification of confusion or lack of participation, to form an opinion about successful learning of concepts.

Metacognitive framework. Garofalo and Lester's (1985) cognitive-metacognitive framework was a useful tool for identifying the students' comments and behavior. Categorizing observations according to orientation, organization, execution, and verification organized the data across the data sources (students), methods (interview types), and data types (cognitive-metacognitive framework categories) for triangulation across the four themes taken from literature. The framework provided a tool for making this study scientific.

This study provided data consistent with two areas of research: metacognition within mathematical problem solving and misconceptions stochastics students hold to be true. It adds to the literature because metacognition, problem solving, and stochastics education were examined together. It is still not clear as to what types of students benefit the most from metacognition. A fourth variable of affect and/or beliefs and success in learning probability and statistics concepts within a metacognitive framework would be a viable next step.

Limitations of the Study

Two major limitations of this study were generalizability and definition of successful. First, not all students were interviewed which eliminates inference onto the population of statistics students, in and out of the classroom. The results of this study only describe what was observed during a ten week period within the course environment. Second, midterm exams were not analyzed. The students' errors on the exams were not identified; were they simple arithmetic errors or was it lack of understanding calculated answers in the problems? To improve this study a closer look at answers to the exam questions is warranted. This would identify if assessments were based on conceptual understanding or only on ability to recall and follow appropriate algorithms.

Future Research

Do instructors in Elementary Probability and Statistics courses view conceptual understanding as important for success in their courses? If developing critical thinking in college students is an ideal, it is important to develop the conceptual understanding of probability and statistics students. Future studies about metacognition and statistics students might include surveying instructors to identify their beliefs.

It would also be beneficial to see if students who do not perform well on assessments and are directed to think about their own learning processes improve their understanding of elementary probability and statistics concepts. A curriculum that explicitly guides all the students in the course to use metacognitive thoughts while learning to solve statistical problems would be in line with Schoenfeld's (1987) ideas about mathematical problem solving. The classroom experience for the students in this study was solving sample problems appropriate to the statistical concepts described in the textbook; metacognitive questions were not modeled. Perhaps a course that is designed to improve metacognition for students who were not successful at their first attempt to learn elementary probability and statistics concepts would provide informative data.

References

Adibnia, A. & Putt, I.J. (1998). Teaching problem solving to year 6 students: A new approach. *Mathematics Education Research Journal, 10*(3), 42-58 Dec 1998.

Ainley, J., & Pratt, D. (2001). Introducing a special issue on constructing meanings from data. *Educational Studies in Mathematics, 45*, 1-8.

Anthony, G. (1996). Active learning in a constructivist framework. *Educational Studies in Mathematics, 31*(4), 349-369.

Batanero, C., Godino, J. D., Vallecillos, A., Green, D. R., & Homes, P. (1994). Errors and difficulties in understanding elementary statistical concepts. *International Journal of Mathematical Education in Science and Technology, 25*(4), 527-547.

Becker, B. J. (1996). A look at the literature (and other resources) on teaching statistics. *Journal of Educational and Behavioral Statistics, 21*(1), 71-90.

Beitz, J. M., & Wolf, Z. R. (1997). Creative strategies for teaching statistical concepts in nursing education. *Nurse Educator, 22*, 30-34.

Borovcnik, M., & Peard, R. (1996). Probability. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 239-287). Dordrecht, The Netherlands: Kluwer Academic Publishers Group.

Brown, A. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. E. Weinert & R. H. Kluwe, (Eds.) *Metacognition, motivation, and understanding* (pp. 65-116). Hillsdale, NJ: Lawrence Erlbaum.

Campione, J. C., Brown, A. L., & Connell, M. L. (1989). Metacognition: On the importance of understanding what you are doing. In R. I. Charles & E. A. Silver

(Eds.), *The teaching and assessing of mathematical problem solving* (pp. 93-114). Reston, VA: Lawrence Erlbaum Associates.

Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning, 1*(1), 5-43.

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *The American Mathematical Monthly, 104*(9), 801-823.

Cohen, D. (Ed.) (1995). *Crossroads in mathematics: Standards for introductory college mathematics before calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges.

Descartes, R. (1952). Rules for the direction of the mind (E.S. Haldane and G.R.I. Ross, Trans.). In R. M. Hutchins & M. J. Adler (Eds.), *Great Books of the Western World* (Vol. 31, pp. 41-67). Chicago, IL: Encyclopedia Britannica, Inc.

Derry, S. J., Levin, J. R., Osana, H. P., Jones, M. S., & Peterson, M. (2000). Fostering students' statistical and scientific thinking: Lessons learned from an Innovative college course. *American Educational Research Journal, 37*(3), 747-773.

Ericsson, K.A. & Simon, H.A. (1993). *Protocol analysis: Verbal reports as data*. Cambridge, MA: The MIT Press.

Ernest, P. (1996). Varieties of constructivism: A framework for comparison. In L. P. Steffe & P. Nesher, (Eds.), *Theories of mathematical learning* (pp. 335-350). Mahwah, NJ: Lawrence Erlbaum.

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children*. Dordrecht, Holland: D. Reidel Publishing Company.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231-235). Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. H. (1987). Speculations about the nature and development of metacognition. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. 21-29). Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. H., Friedrichs, A. G., & Hoyt, J. D. (1970). Developmental changes in memorization processes. *Cognitive Psychology, 1*, 324-340.

Galagedera, D., Woodward, G., Degamboda, S. (2000). An investigation of how perceptions of mathematics ability can affect elementary statistics performance. *International Journal of Mathematical Education in Science & Technology, 31*(5), 679-689.

Garofalo, J., & Lester, F. K., Jr. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*, 163-176.

Garfield, J., & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education, 19*, 44-63.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative Zones of Proximal Development in small group problem solving. *Educational Studies in Mathematics, 49*(2), 193-223.

Greer, B. (1996). Theories of mathematics education: The role of cognitive analysis. In L. P. Steffe & P. Nesher, (Eds.), *Theories of mathematical learning* (pp. 179-196). Mahwah, NJ: Lawrence Erlbaum.

Hansen, R. S., McCann, J., & Myers, J. L. (1985). Rote versus conceptual emphases in teaching elementary probability. *Journal for Research in Mathematics Education, 16*(5), 364-374.

Hatano, G. (1996). A conception of knowledge acquisition and its implications for mathematics education. In L. P. Steffe & P. Nesher (Eds.), *Theories of mathematical learning* (pp. 197-217). Mahwah, NJ: Lawrence Erlbaum.

Kahneman, D., Slovic, P., & Tversky, A. (Eds.). (1982). *Judgment under uncertainty: Heuristics and biases*. New York: Cambridge University Press.

Kahneman, D., & Tversky, A. (1982). On the study of statistical intuitions. *Cognition, 11*, 123-141.

Kirkpatrick, J. (Ed.). (1985). A retrospective account of the past 25 years of research on teaching mathematical problem solving. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 1-15). Hillsdale, NJ: Lawrence Erlbaum.

Kluwe, R. H. (1987). Executive decisions and regulation of problem solving behavior. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. 31-64). Hillsdale, NJ: Lawrence Erlbaum.

Lester, F. K. (1985). Methodological considerations in research on mathematical problem-solving instruction. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 41-69). Hillsdale, NJ: Lawrence Erlbaum.

Lester, F. K., Jr. (1989). Reflections about mathematical problem-solving research. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical* *problem solving* (pp. 115-124). Reston, VA: Lawrence Erlbaum and the National Council of Teachers of Mathematics.

Lester, F. K., Jr. (1994). Musings about problem-solving research: 1970-1994. *Journal of Research in Mathematics Education, 25*, 660-675.

Lester, F. K., Jr., Garofalo, J., & Kroll, D. L. (1989). The role of metacognition in mathematical problem solving: A study of two grade seven classes (Report No. NSF-MDR-85-50346). Bloomington, IN: Indiana University. (ERIC Document Reproduction Service No. ED 314 255)

Loosen, F., Lioen, M., & Lacante, M. (1985). The standard deviation: Some drawbacks of an intuitive approach. *Teaching Statistics, 7*(1), 2-5.

MacClain, K., & Cobb, P. (2001). Supporting students' ability to reason about data. *Educational Studies in Mathematics, 45*, 103-129.

Maqsud, M. (1998). Effects of metacognitive instruction on mathematics achievement and attitude towards mathematics of low mathematics achievers. *Educational Research,40*(2), 237-243.

Mathematical Association of America. (1998*). Quantitative reasoning for college graduates: A complement to the standards*. Retrieved June 21, 2004 from http://www.maa.org/past/ql/ql_toc.html

Mevarech, Z.R. (1999). Effects of metacognitive training embedded in cooperative settings on mathematical problem solving. *Journal of Educational Research, 92*(4), 195-205.

Miles, M.B. & Huberman, A.M. (1994). *An expanded sourcebook: Qualitative data analysis* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Minsky, M. (Ed.). (1968). *Semantic information processing*. Cambridge, MA: The MIT Press.

Moore, D. S., & McCabe, G. P. (2003). *Introduction to the practice of statistics* (4th ed.). New York: W. H. Freeman and Company.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics in the 1980s*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

Onwuegbuzie, A.J. (2000). Attitudes toward statistics assessments. *Assessment & Evaluation in Higher Education, 25*(4), 321-340. Retrieved November 16, 2001 from http://search.epnet.com/login.aspx?direct=true&AuthType=cookie,ip,url,uid&db=slh&an=3990965

Patton, M.Q. (1990). *Qualitative evaluation and research methods* (2nd ed.). Newbury Park, CA: Sage Publications.

Pfannkuch, M. & Brown, C.M. (1996). Building on and challenging students' intuitions about probability: Can we improve undergraduate learning? [Electronic version]. *Journal of Statistics Education, 4*(1). Retrieved September15, 2004 from http://www.amstat.org/publications/jse/v4n1/ pfannkuch.html

Pollatsek, A., Lima, S., & Well, A.D. (1981). Concept or computation: Students' understanding of the mean. *Educational Studies in Mathematics, 12*, 191-204.

Pugalee, D.K. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics, 101*(5), 236-245.

Pólya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Garden City, NY: Doubleday Anchor Books.

Schoenfeld, A. H. (1979). Teaching problem solving in college mathematics: The elements of a theory and a report on the teaching of general mathematical problems-solving skills. In R. Lesh & D. Mierkiewicz & M. Kantowski (Eds.), *Applied mathematical problem solving* (pp. 33-71). Columbus, OH: The Ohio State University.

Schoenfeld, A. H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 345-395). New York: Academic Press.

Schoenfeld, A. H. (1985). Metacognitive and epistemological issues in mathematical understanding. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 361-380). Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science in mathematics education* (pp. 189-215). Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan Publishing Company.

Shaughnessy, J. M. (1985). Problem-solving derailers: The influence of misconceptions on problem solving performance. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 399-416). Hillsdale, NJ: Lawrence Erlbaum.

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465-493). New York: MacMillan.

Silver, E. A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 247-266). Hillsdale, NJ: Lawrence Erlbaum.

Sternberg, R. J. (1982). A componential approach to intellectual development. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 1, pp. 413-463). Hillsdale, NJ: Erlbaum.

Thorndike, E. L. (1917). Reading as reasoning: A study of mistakes in paragraph reading. *The Journal of Educational Psychology, 8*(6), 323-332.

Thorndike, E. L. (1924). Mental discipline in high school studies. *The Journal of Educational Psychology, 15*, 1-98.

Tukey, J. W. (1962). The future of data analysis. *Annals of Mathematical Statistics, 33*(1), 1-67.

Tukey, J. W. (1977). *Exploratory data analysis*. Reading, MA: Addison-Wesley.

Tversky, A., & Kahneman, D. (1971). Belief in the law of small numbers. *Psychological Bulletin, 76*(2), 105-110.

von Glassersfeld, E. (1996). Aspects of radical constructivism and its educational recommendations. In L. P. Steffe & P. Nesher, (Eds.), *Theories of mathematical learning* (pp. 307-314). Mahwah, NJ: Lawrence Erlbaum.

Weinert, F. E. (1987). Introduction and overview: Metacognition and motivation as determinants of effective learning and understanding. In F. E. Weinert & R. H. Kluwe, (Eds.) *Metacognition, motivation, and understanding* (pp. 1-19). Hillsdale, NJ: Lawrence Erlbaum.

Well, A. D., Pollatsek, A., & Boyce, S. J. (1990). Understanding the effects of sample size on the variability of the mean. *Organizational Behavior and Human Decision Processes, 47*, 289-312.

Zan, R. (2000). A metacognitive intervention in mathematics at university Level. *International Journal of Mathematical Education in Science and Technology, 31*(1), 143-150.

Appendix A

Consent Form

University of Cincinnati
Consent to Participate in a Research Study
College of Education/Curriculum and Instruction
Teri Rysz
242-9420/wheeleti@email.uc.edu

Title of Study: Metacognition in Learning Elementary Probability and Statistics

Introduction: Before agreeing to participate in this study, it is important that the following explanation of the proposed procedures be read and understood. It describes the purpose, procedures, risks, and benefits of the study. It also describes the right to withdraw from the study at any time. It is important to understand that no guarantee or assurance can be made as to the results of the study.

Purpose: As a portion of the requirements for a Doctor of Education Degree at the University of Cincinnati, I am required to research an area in curriculum and instruction in mathematics. The purpose of this study is to better understand thought processes of the students enrolled in Elementary Probability and Statistics. I will be collecting data through observations of just the students enrolled in this section of the course; you will be one of approximately 50 participants taking part in this study.

Duration: Your participation in this study will be for the remainder of this spring quarter.

Procedures: During the course of this study, the following will occur:

- For the remainder of this quarter, while your instructor conducts class, I will be observing the students, taking notes on behavior and comments regarding thought processes while learning probability and statistics.
- In order to analyze results of student thought processes more thoroughly, the researcher will access all the participating students' grades on the first midterm and their final grades for this quarter of statistics. No other grades will be given to me.
- After the first midterm has been returned to the students, I will ask six students to participate in three activities outside the classroom. Students who are not asked to participate outside of class will only be observed in the class for the remainder of the quarter. The out of class activities follow:
  1. The first activity is called an individual interview and will last 30 to 60 minutes. It will take place in 809D Old Chemistry. The student will be asked to solve a statistical problem out loud and the activity will be audiotaped.
  2. The second activity is another individual interview in which I will make a copy of your notes from one class session and read through them with you immediately following that class. This will also last approximately 30 to 60 minutes and will take place in 809D Old Chemistry. I will ask the student questions such as "Why did you write this?" or "What were you thinking about when you wrote this?" This activity will also be audiotaped.
  3. The third activity will be a group problem solving session that will be held in 607 Teachers College at a time convenient to the students who are invited to participate. I will ask the six students to spend approximately 30 minutes solving a statistical problem as a group. This activity will be videotaped and audiotaped.

Exclusion: You will not be able to be a part of my study if you are not yet 18 years old. You may still be a part of the class, but you will need to speak to me if you are younger than 18.

Risks/Discomforts: There are no foreseeable risks or discomforts anticipated with this study.

Benefits: Responses to questions will in no way affect your grade; participation will not add to nor take away from grade points. Your course will not be different from other sections of this statistics course because of this study. You will receive no direct benefit from your participation in this study, but your participation may help statistics instructors and educators, because we will better understand how students learn probability and statistics.

Alternatives: If you choose not to participate in this study, you will still be expected to complete all course requirements, and you will not be included in the observations nor in the out of class activities.

Confidentiality: Every effort will be made to maintain the confidentiality of your study records. All collected data will remain confidential and will not be traceable to any individual student. The instructor of this course will not see any of the collected data until the final report is submitted and at that time will only have access to whole class (aggregate) data. After data for the class as a whole are submitted as part of my research all written records and tapes will be destroyed. Agents of the University of Cincinnati will be allowed to inspect sections of the research records related to this study. The data from the study may be published; however, you will not be identified by name. Your identity will remain confidential unless disclosure is required by law, such as mandatory reporting of child abuse, elder abuse, or immediate danger to self or others.

Payments to participants: Students who complete all 3 interviews outside class will receive $15.00 ($5.00 for each interview) following the third interview. No payment will be made to a student who is only observed in class nor to any student who does not complete all 3 of the out-of-class activities.

Right to refuse or withdraw: Your participation is voluntary and you may refuse to participate, or may discontinue participation AT ANY TIME during the quarter—without difficulty, undue embarrassment, or negative consequences—by informing me in writing. The investigator has the right to withdraw you from the study AT ANY TIME. Your withdrawal from the study may be for reasons related solely to you (for example, not following study-related directions from the investigator, etc.) or because the entire study has been terminated.

Questions: If you have questions or comments about this study, you may call me, Teri Rysz, at 242-9420. In addition, if you have any problems, questions, or concerns that I do not adequately address, you may contact my academic advisor, Dr. Janet Bobango, at 556-3569 or the College of Education Department Head, Dr. Glenn Markle at 556-3582. If you have any questions about your rights as a research participant, you may call Dr. Margaret Miller, Chair of the Institutional Review Board—Social and Behavioral Sciences, at 513-558-5784.

Legal Rights: Nothing in this consent form waives any legal right you may have nor does it release the investigator, the institution, or its agents from liability for negligence.

_______________________________________ __________________

Principal Investigator Signature **Date**

I HAVE READ THE INFORMATION PROVIDED ABOVE. I VOLUNTARILY AGREE TO PARTICIPATE IN THIS STUDY. I WILL RECEIVE A COPY OF THIS CONSENT FORM FOR MY INFORMATION.

_______________________________________ __________________

Participant Signature **Date**

Appendix B

Classroom Organizer of Students' Indications of Metacognition

Date_______________

Orientation
Organization
Execution
Verification
Other

Appendix C

Examples of Classroom Organizer of Students' Indications of Metacognition

Classroom organizer of students' indications of metacognition   Date 4-10-03

<u>Orientation: Strategic behavior to assess and understand a problem</u>

Students copying information from the board. All the Students are very quiet, listening to T

Students' heads are nodding in response to Teacher's yes/no question

*2 Students sharing calculator—make comments to each other once in a while—listen to T more

Student asked "What do the numbers in the range mean?"

*Student stood up and adjusted camera light not to be in her eyes.

Teacher asked "What did you get?" Student said "1067.461" Teacher asked "Do you round up or round down?" Student said "No, round down." Then quickly said, "Yes, round up." Another Student asked "Round up because can't have fraction of a person?" [The correct answer was round up to keep the margin of error at a minimum level but this was not explained to the Student who was struggling to understand why she should round up when the tenths place was a 4. This is opposite to what is taught about rounding in grade school.]

Student asked "How do you know if we use a 95% confidence interval?"

*Student requested "Walk us through where to go in the calculator again" [to get p value for proportion H_0.]

<u>Organization: Planning of behavior and choice of actions</u>

Many Students pulled out TI83 calculators when asked to calculate the confidence interval.

*Student stood up and adjusted camera light not to be in her eyes.

<u>Execution: Regulation of behavior to conform to plans</u>

Teacher asked "What number do we multiply by to get a 95% confidence interval?" One Student answered 1.96 [which was correct.]

*2 Students sharing calculator—make comments to each other once in a while—listen to T more

*Teacher assigned: Find 95% confidence interval for x = 32 and n = 95. 1 Student found answer and replied (.242, .432). Teacher asked "Does anyone agree with that?" Several Students said "Yeah." Student answered ".095" to Teacher's question

<u>Verification: Evaluation of decisions made and of outcomes of executed plans</u>

*2 Students sharing calculator—make comments to each other once in a while—listen to T more

Student commented politely "Wouldn't it be 54?" [instead of 52 that T wrote on the board]

Teacher assigned: Find 95% confidence interval for x = 32 and n = 95. 1 Student found answer and replied (.242, .432). Teacher asked "Does anyone agree with that?" Several Students said "Yeah."

*Student requested "Walk us through where to go in the calculator again" [to get p value for proportion H_0.]

<u>Other</u>

Student sleeping

Teacher said "Use your noodle a little bit." Only 1 Student responded with a giggle. Only one that noticed? Or the only one who thought it was a funny thing to say?

Students are reluctant to publicly vote for answer to "How many would reject H_0?"

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 4-15-03

<u>Orientation: Strategic behavior to assess and understand a problem</u>

27 people present when class begins. Teacher counts them and then asks who has a cell phone, 20 people raised their hands n = 27, x = 20

*2 Students whispering as Teacher and Student work through problem

*Teacher said, "Look in your table to see if it agrees" A few people started looking in the book for the table.

*Teacher said, "I goofed up here. Do you understand how this works?" One Student had a definite "Yes." She pays attention all through class. [Is there metacognition here?]

*2 Students still discussing last problem

<u>Organization: Planning of behavior and choice of actions</u>

none

<u>Execution: Regulation of behavior to conform to plans</u>

Student responds to Teacher's question ".575" Teacher says "Is that what you get?" Student says "Yep" Teacher asks "Which selection is that?" Student responds "B"

*2 Students whispering as Teacher and Student work through problem

*Some Students working on calculators. Some just staring.

*Teacher said, "Look in your table to see if it agrees" A few people started looking in the book for the table.

Most Students working problems

<u>Verification: Evaluation of decisions made and of outcomes of executed plans</u>

*Teacher asked, "How many think this is right?" "How many think this is wrong?" Not much response from the Students. Teacher continued "How many are not thinking?" Several Students raised their hand. Teacher repeated the questioning and more Students participated in the vote. [lack of metacognition? When behavior was pointed out, Students cooperated/participated more.]

*Teacher said, "Look in your table to see if it agrees" A few people started looking in the book for the table.

*Teacher said, "I goofed up here. Do you understand how this works?" One Student had a definite "Yes." She pays attention all through class. [Is there metacognition here?]

*2 Students still discussing last problem—orientation

<u>Other</u>

Some Students working on calculators. Some just staring.

*Teacher asked, "How many think this is right?" "How many think this is wrong?" Not much response from the Students. Teacher continued "How many are not thinking?" Several Students raised their hand. Teacher repeated the questioning and more Students participated in the vote. [lack of metacognition? When behavior was pointed out, Students cooperated/participated more.]

2 Students left at 11:35

No sleepers yet

1 Student left at 11:47

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 4-17-03

Orientation: Strategic behavior to assess and understand a problem

Teacher asked "How many got a perfect score?" [exercise in just guessing answers in multiple choice] 10 Students raised hand (out of 35?)

*Student turns to another Student and says "If in critical region then reject H_0" Other Student responds "Yeah, I think so."

Another couple that works together paying attention and sharing with each other

Jessica and girl in front row are responding to Teacher's questions. They answer many of Teacher's rhetorical questions. Do they need to do this to keep focused on learning?

Student raised hand "After you found 007 on the chart what did you do after that?" [orientation] Student is sitting forward in his seat [to better hear/understand?]

Another Student question waiting with hand up. "Depending on what p value is, we're going to reject, right?" "And then . . ." [interrupted] "Are you going to have 1 or 2 answers on there?" "Yeah, it depends on if it's 1 or 2 tailed." I wonder if her question is being answered.

Organization: Planning of behavior and choice of actions

none

Execution: Regulation of behavior to conform to plans

*1 Student working on calculator as Teacher works on board with formulas, 5 minutes later playing a game on TI83

Most Students have TI83—they are using them after Teacher directed "calculate this to 3 decimal places."

*Teacher asked "Which selection is it?" Student responded "6" Students ahead of Teacher in working out a problem. Teacher asked "What z value do you see?" Student "-2.447" Teacher "and the p value?" Student said ".014"

Verification: Evaluation of decisions made and of outcomes of executed plans

*Student turns to another Student and says "If in critical region then reject H_0" Other Student responds "Yeah, I think so."

*Teacher asked "Which selection is it?" Student responded "6" Students ahead of Teacher in working out a problem. Teacher asked "What z value do you see?" Student "-2.447" Teacher "and the p value?" Student said ".014"

Other

*1 Student working on calculator as Teacher works on board with formulas, 5 minutes later playing a game on TI83

Teacher says, "Can do this on a calculator you get in a Cornflake box." Student laughed quietly

Student who sleeps usually has written nothing down for notes—I'd like to interview him—maybe he'll be one of the quartiles.

One couple that usually works together—both are absent today. [this ended up being one of my interviews.]

Teacher demonstrated how to use the CD that comes with the text to do practice quizzes.

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 4-24-03

<u>Orientation: Strategic behavior to assess and understand a problem</u>
Most Students taking notes about left handed and right handed batters
1 Student answering all calculation questions from Teacher
Most Students still writing down notes—finding conditional distributions
*More people are calling out answers to percentage/proportion questions now.
*Sri answering calculation questions. He sits in the first row in the middle when he is in class.
1 Student with a question
<u>Organization: Planning of behavior and choice of actions</u>
*Teacher directed Students to find xbar of 7 numbers on board. A few Students took out calculators. Teacher asked, "What proportion of these numbers are odd?" Lots of blank looks from the students.
Students writing down formula for expected value
<u>Execution: Regulation of behavior to conform to plans</u>
*More people are calling out answers to percentage/proportion questions now.
<u>Verification: Evaluation of decisions made and of outcomes of executed plans</u>
*More people are calling out answers to percentage/proportion questions now.
<u>Other</u>
A suggestion was made to check ebay for buying TI83s
*Teacher directed Students to find xbar of 7 numbers on board. A few Students took out calculators. Teacher asked, "What proportion of these numbers are odd?" Lots of blank looks from the students.
2 Students chatting a lot about other things (not proportions)
Jessica isn't here
Couple in back still chatting about other than stats
Student answering calculation questions. He sits in the first row in the middle when he is in class.

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 4-29-03

Orientation: Strategic behavior to assess and understand a problem
Lots of disagreement about χ^2 arithmetic, most of class discussing now.
Some Students looking in textbook for Table F [I assume]
only a few Students answer Teacher's questions. The same few keep nodding
My personal observation is Students who sit in middle front communicate with Teacher more. Is this metacognition at work?
One Student who hasn't answered any questions answered when I moved behind him. Another previously silent Student answered a question after I moved here.
1 Student helping Student who is confused
Organization: Planning of behavior and choice of actions
*Teacher gave problem with a 3x4 matrix. Most Students working with calculators [Teacher organization/Student execution]
Execution: Regulation of behavior to conform to plans
Teacher directed Students to find expected value. Some did it, more did not have calculator [teacher-made plans]
Teacher had to blatantly point at Students to get them to find an expected value—a very simple arithmetic problem $\frac{rowtotal \times columntotal}{grandtotal}$
New problem—more people answering same questions as previous problem, more confidence? Almost everyone doing calculations now
One Student raised hand with answer to Teacher's question (male)
One Student raised hand with answer to Teacher's question (male)
One Student raised hand with answer to Teacher's question (female)
One Student raised hand with answer to Teacher's question (female)
Perhaps this is a problem the Students have more interest in finding an answer.
I hear whistles from Students when they found χ^2 on TI. It's so much easier than doing all the calculations.
*Teacher gave problem with a 3x4 matrix. Most Students working with calculators [Teacher organization/Student execution]
Verification: Evaluation of decisions made and of outcomes of executed plans
Key is on Blackboard and on electronic reserves
Student found error written on board—an answer given by another Student 5 minutes earlier
Other
Students are very quiet, depressed over exam grade?
1 girl sleeping
1 girl looking at newspaper ads
1 Student just staring, but in front row. No calculator, no text
1 girl writing in appointment calendar.
1 Student in back row looking at small book. I had to walk by to see what he was looking at, it's a Spanish book. He has several Spanish books on his desk.
Teacher invited people without TI83 to leave, and absolutely no one left

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 5-1-03

<u>Orientation: Strategic behavior to assess and understand a problem</u>

Mark. is writing a lot—very engaged in communication with Teacher. Can't see Charlene's desk.

Teacher—"Raise your hand if you know how to deduce how many don't have cell phones." 3 Students raised their hands very low.

Teacher asked "What's the relationship between t and χ^2?" Mark answered correctly.

<u>Organization: Planning of behavior and choice of actions</u>

People using calculators to calculate χ^2 from raw data

1 Student brave enough to answer direct question "What would be the smart thing to do if you got this question on a test?"

<u>Execution: Regulation of behavior to conform to plans</u>

Students with TI83 plugging in numbers after Teacher suggested working problem

<u>Verification: Evaluation of decisions made and of outcomes of executed plans</u>

none

<u>Other</u>

Charlene sits with the same person every class right in the middle of the room (5 rows up (3 rows behind them)

Mark sits by himself to the right of the middle section 4 rows up

One Student not writing anything, why not? Later: same guy—just observing everything

Student who arrived ten minutes late is now going out of the room, leaving books on the desk. Later: Here comes Student that arrived late and left room. She's looking at her midterm, using her calculator but doesn't appear to be about today's lecture.

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 5-6-03

Orientation: Strategic behavior to assess and understand a problem

Student asked about another drawing on the board—teacher hadn't reached that point of lecture yet

Student adjusted computer lamp again

*Most students working on calculators, following teacher's work at board

Copying down formulas from screen

*11:30 Teacher said "OK—that's a quick review—now you ask questions."

Organization: Planning of behavior and choice of actions

Before class: two students discussing χ^2 before class. One said "Let me write that down. He'll probably ask something like that."

Teacher divided up class to find four contributions to χ^2 statistic

	X	Y	Total
A	19 (18.3)	22 (22.7)	41
B	39 (39.7)	50 (49.3)	89
Total	58	72	130

Teacher asked "Anybody remember the other way to do this?"

Student replied "Get the z score and square it."

Teacher described a little more precisely

Student asked "Is the test in two parts?"

Student asked "Are you still doing final in two parts?"

Student asked "Is Simpson's Paradox going to be on the test?"

Student asked "What portion of the test do you want us to do without the calculator?"

Student asked "Can you do one example problem with the chart?"

Student asked "Does this test just cover χ^2?"

Student asked "If we put Ho are they related and the other one independent or are you going to take off?" [model]

Student asked "Do we need to know how to do marginal and conditional distributions?"

Execution: Regulation of behavior to conform to plans

Before class: Two students talking about how to do χ^2 on TI83

Before class: Three other students talking about how to do χ^2 on TI83

*Most students working on calculators, following teacher's work at board

Verification: Evaluation of decisions made and of outcomes of executed plans

Student asked other student why her calculator showed a completely different answer. Other student found her input error

Other

Several (5) students arrived up to ten minutes late

*11:30 Teacher said "OK—that's a quick review—now you ask questions."

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 5-13-03

<u>Orientation: Strategic behavior to assess and understand a problem</u>
Maggie taking notes [good]
Teacher said "without a table of values see if you can sketch this: $y = 3x - 2$." [on overhead]
Teacher said "I teach students this way so you'll have a mental image as soon as you see the equation."
Two students wrote down "line of best fit"
<u>Organization: Planning of behavior and choice of actions</u>
none
<u>Execution: Regulation of behavior to conform to plans</u>
Teacher said "Finish up table on scratch paper." And most students working. Some students giving y coordinate.
Four students did not get r so Teacher showing how to turn diagnostic on
<u>Verification: Evaluation of decisions made and of outcomes of executed plans</u>
key on Blackboard this afternoon
Teacher asked "Do you notice a pattern?"
<u>Other</u>
none

Classroom organizer of students' indications of metacognition Date 5-15-03

Orientation: Strategic behavior to assess and understand a problem

One student reading magazine, taking notes now

One student just staring—no notes, no calculator. I don't know his name but he was studying Spanish one day. He's reading something (book) now.

Maggie's interview is today—she's paying attention

*Everyone is very quiet today. Seem to be paying attention—I'm fairly bored watching them.

Organization: Planning of behavior and choice of actions

none

Execution: Regulation of behavior to conform to plans

none

Verification: Evaluation of decisions made and of outcomes of executed plans

none

Other

*Everyone is very quiet today. Seem to be paying attention—I'm fairly bored watching them.

The student who usually just sleeps has a notebook open and a pen in his hand. Is it because I'm sitting very close to him today? I usually sit in the front of the classroom but am in the middle today. He hasn't written anything down yet (11:40 a.m.)

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 5-20-03

<u>Orientation: Strategic behavior to assess and understand a problem</u>
Attendance seems higher than usual
*Mark only one answering teacher's table-reading questions
Most students appear to be paying attention
Haven't noticed Jessica taking notes—she sits second row middle chair. Haven't noticed her absent at all.
Working on TI83—students entering data. Just a few not doing it (back row).
*Lots of students answering teacher's questions about result on calculator
Active participation, more students sharing with each other
<u>Organization: Planning of behavior and choice of actions</u>
none
<u>Execution: Regulation of behavior to conform to plans</u>
Some students (4) entering raw data even though not instructed to do so.
<u>Verification: Evaluation of decisions made and of outcomes of executed plans</u>
Mark answered teacher's question "What's another way we'll get a large quotient for F?" with "A small denominator" fairly loudly. Confident? He was correct.
*Mark only one answering teacher's table-reading questions
*Lots of students answering teacher's questions about result on calculator
<u>Other</u>
Do these students understand or are they so lost they are dumbfounded?
Student who studies Spanish is reading a small book not Stats
Charlene is helping her friend
One student checking phone, laying head down, yawning, not taking notes. Wonder how she's doing.
The student who was angry about my observations is just watching even though he has a TI83 on his desk.
Only a few students have used Excel in another course

*Indicates categorized in more than one category

Classroom organizer of students' indications of metacognition Date 5-22-03

Orientation: Strategic behavior to assess and understand a problem

Excel explanation—all students watching

Charlene taking notes—unusual

All seven of my interviewees are here. They all seem to have very good attendance.

Organization: Planning of behavior and choice of actions

Having fun with Excel—students seem to enjoy

One student tried to help teacher see print preview which doesn't work without a printer

Execution: Regulation of behavior to conform to plans

none

Verification: Evaluation of decisions made and of outcomes of executed plans

none

Other

Some of the students have used Chart Wizard in Excel before

This is the last class I'll be observing. The next one is cancelled so we'll be doing the group session. Then the third test and the alternative final exam the last week of class.

*Indicates categorized in more than one category

Appendix D

Data Analysis Problem for Individual Interviews

Investment theory uses the standard deviation of returns to describe the volatility or risk of an investment. To describe how the risk of a specific security is related to that of the market as a whole, we use least-squares regression. The plot on page 214 of the text shows the monthly percent total return *y*, Philip Morris common stock, against the monthly return *x*, the Standard & Poor's 500 stock index. The data are from the market for the period between July 1990 and June 1997. The one clear outlier turns out not to be very influential. Here are the basic descriptive measures:

$\bar{x} = 1.304 \quad s_x = 3.392$

$\bar{y} = 1.878 \quad s_y = 7.554 \quad r = 0.5251$

a) Find the equation of the least-squares line ($y = a + bx$) using the basic descriptive measures above and the equations for finding slope and intercept ($b = r \times \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$).

b) What percent of the variation (r^2) in Philip Morris stock (y) is explained by the linear regression equation of *y* with the market as a whole (x)?

c) Explain carefully what the slope of the line tells us about how Philip Morris stock responds to changes in the market. This slope is called "beta" in investment theory.

d) Returns on most individual stocks have a positive correlation with returns on the entire market. That is, when the market goes up, an individual stock tends to also go up. Explain why an investor should prefer stocks with beta > 1 when the market is rising and stocks with beta < 1 when the market is falling. (Moore & McCabe, 2003, pp. 213-214)

Solutions to
Data Analysis Problem for Individual Interviews

Investment theory uses the standard deviation of returns to describe the volatility or risk of an investment. To describe how the risk of a specific security is related to that of the market as a whole, we use least-squares regression. The plot on page 214 of the text shows the monthly percent total return *y*, Philip Morris common stock, against the monthly return *x*, the Standard & Poor's 500 stock index. The data are from the market for the period between July 1990 and June 1997. The one clear outlier turns out not to be very influential. Here are the basic descriptive measures:

$\bar{x} = 1.304$ $s_x = 3.392$

$\bar{y} = 1.878$ $s_y = 7.554$ $r = 0.5251$

a) Find the equation of the least-squares line ($y = a + bx$) using the basic descriptive measures above and the equations for finding slope and intercept ($b = r \times \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$).

$b = 0.5251 \times (7.554 \div 3.392) = 1.1694$

$a = 1.878 - (1.1694 \times 1.304) = 0.3531$

The equation for the least squares line then is $y = 0.3531 + 1.1694x$

b) What percent of the variation (r^2) in Philip Morris stock (y) is explained by the linear regression equation of y with the market as a whole (x)?

$r = 0.5251$ and $r^2 = 0.5251^2 = 2757$

Therefore the percent of variation is 27.57%.

c) Explain carefully what the slope of the line tells us about how Philip Morris stock responds to changes in the market. This slope is called "beta" in investment theory.

For every percentage point rise in the overall market Philip Morris stock rises 1.1694 percentage points.

d) Returns on most individual stocks have a positive correlation with returns on the entire market. That is, when the market goes up, an individual stock tends to also go up. Explain why an investor should prefer stocks with beta > 1 when the market is rising and stocks with beta < 1 when the market is falling. (Moore & McCabe, 2003, pp. 213-214)

As the overall market is rising the stockholder would like his/her stock to rise more than the overall market, beta > 1. When the market is falling overall the individual stock holder prefers to loose less than the overall market, beta < 1.

Appendix E

Data Analysis Problem for Group Interviews

The U.S. Agency for International Development provides tons of corn soy blend (CSB) for development programs and emergency relief in countries throughout the world every year. CSB is a highly nutritious, low-cost fortified food that is partially precooked and can be incorporated into different food preparations by the recipients. As part of a study to evaluate appropriate vitamin C levels in this commodity, measurements were taken on samples of CSB produced in a factory. The following data are the amounts of vitamin C, measured in milligrams per 100 grams (mg/100 g) of blend (dry basis), for a random sample of size 8 from one production run:

41 41 38 37 26 37 29 38

We want to find a 90% confidence interval for μ, the mean vitamin C content of the CSB produced during this run.

a) The sample mean is

b) The standard error is

c) What is the critical value needed for this 90% confidence interval?

d) The 90% confidence interval for μ is

The specifications for production are designed to produce a mean μ vitamin C content of 40 mg/100 g of CSB in the final product.

e) State the appropriate hypotheses to determine if the sample is different from the specified level of vitamin C.

f) Calculate the appropriate standardized test statistic.

g) What is the *p*-value or range of *p*-values for this test?

h) What do you conclude and why?

i) Does your conclusion support or contradict the answer found in part d above?

Solutions to
Data Analysis Problem for Group Interviews

The U.S. Agency for International Development provides tons of corn soy blend (CSB) for development programs and emergency relief in countries throughout the world every year. CSB is a highly nutritious, low-cost fortified food that is partially precooked and can be incorporated into different food preparations by the recipients. As part of a study to evaluate appropriate vitamin C levels in this commodity, measurements were taken on samples of CSB produced in a factory. The following data are the amounts of vitamin C, measured in milligrams per 100 grams (mg/100 g) of blend (dry basis), for a random sample of size 8 from one production run:

41 41 38 37 26 37 29 38

We want to find a 90% confidence interval for μ, the mean vitamin C content of the CSB produced during this run.

a) The sample mean is
   (41 + 41 + 38 + 37 + 26 + 37 + 29 + 38)/8 = 35.8750
b) The standard error is
   Standard deviation = Square root of $((41 – 35.875)^2 + (41 – 35.875)^2 + (38 – 35.875)^2 + (37 – 35.875)^2 + (26 – 35.875)^2 + (37 – 35.875)^2 + (29 – 35.875)^2 + (38 – 35.875)^2)/(8 – 1)$
   = 5.4625
   Standard error = Standard deviation/Square root of the sample size
   = 5.4625/√8 = 1.9313
c) What is the critical value needed for this 90% confidence interval?
   Found in Table D of the textbook, approximately 1.895
d) The 90% confidence interval for μ is
   35.875 ± (1.895 × 1.9313) = 35.875 ± 3.6598 OR (32.2152, 39.5348)

The specifications for production are designed to produce a mean μ vitamin C content of 40 mg/100 g of CSB in the final product.

e) State the appropriate hypotheses to determine if the sample is different from the specified level of vitamin C.
   H_0: $\mu = 40$ H_a: $\mu \neq 40$
f) Calculate the appropriate standardized test statistic.
   t = (35.875 – 40) / 1.9313 = -2.1359
g) What is the p-value or range of p-values for this test?
   If Table D from the textbook is used $0.10 > p > 0.05$
   If the TI83 graphing calculator is used $p = 0.07$
h) What do you conclude and why?
   Either of the following answers is OK as long as appropriate reasoning is given.
   1) Reject H_0 because $p < 0.10$. The vitamin C level differs from 40.
   2) Accept H_0 because $p > 0.05$. The vitamin C level does not differ from 40.
i) Does your conclusion support or contradict the answer found in part d above?
   This answer depends on what answers are in part h and d. Statistically the answers should support each other because an α of 0.10 would be appropriate to use with a 90% confidence interval.