From **Gal, I. & Garfield, J. B. (editors).** <u>**The Assessment Challenge in Statistics Education**</u>. IOS Press, 1997 (on behalf of the ISI). Pages 65-83. ISBN 90 5199 333 1. Copyright holder: International Statistical Institute. Permission granted by ISI to the IASE to make the chapters from this book freely available on the Internet. The References section is a separate chapter and is available on the Internet at http://www.stat.auckland.ac.nz/~iase/publications/assessbkref.

6. Using "Real-Life" Problems to Prompt Students to Construct Conceptual Models for Statistical Reasoning

Richard Lesh
Miriam Amit
Roberta Y. Schorr

Purpose

The purpose of this chapter is to examine a "model-eliciting activity," based upon a "real-life" problem situation, in which students were provided with an opportunity to construct powerful ideas relating to data analysis and statistics, without explicitly being taught. Student results of this activity will be examined that reveal the somewhat surprising fact that children, even those who traditionally do not perform well in mathematics, can invent more powerful ideas relating to trends, averages, and graphical representations of data than their teachers ever anticipated. The student results shared in this chapter are not unique. In classrooms where we have piloted and refined problems (including the one presented), one common observation is that many of the children who emerge as "most productive" are often those whose mathematical abilities had not been recognized or rewarded by their teachers in the past.

INTRODUCTION: MATHEMATICAL MODELS AND MODEL-ELICITING ACTIVITIES

The use of realistic problems to assess student understanding is a recommended practice in statistics education. We have found that when students develop their own ideas about problems using realistic models, learning is enhanced. In order to make sense of the problem activity, and the accompanying student results, it is important to consider what is meant by a mathematical "model" and the characteristics of "model-eliciting activities." A mathematical model can be considered to be a functioning system for describing, explaining, constructing, modifying, manipulating, and predicting a complex series of experiences. Models are organized around situations and experiences, and people interpret problem-solving situations by "mapping" them into their own internal descriptive or explanatory systems (models). Once the given situation has been mapped into an internal model, transformations within the model can take place which can produce a prediction within the modeled situation. This in turn can lead to further predictions, descriptions, or explanations for use back in the problem situation. Models help us to organize relevant information and consider meaningful patterns that can be used to generate or (re)interpret hypotheses about given situations or events, generate explanations of how information is related, and make decisions based on selected cues and information. These internal models develop in stages. Early conceptualizations or models can be fuzzy, or even distorted versions of later models, and several alternative models may be available to interpret a given problem situation. As can be seen in the student results and interpretations that follow, the children went through several "modeling cycles" in which they reinterpreted the givens, goals, and solution paths. They made modifications and refinements to their models during each cycle so that useful predictions, generalizations, and descriptions could be made for the given problem.

The problem activity described in this chapter was designed according to the following six principles, in order to create the need for students to construct, refine, and extend significant mathematical models. These six principles were developed by expert teachers along with mathematics educators and researchers (for a more complete description, see Lesh, Hoover & Kelly, 1992). While these principles might appear to be rather like "common sense," we have found that many of them tend to be violated by virtually every problem that we have seen in major textbooks and tests. Therefore, in some sense, they are quite radical.

The Reality Principle:

Could this really happen in a "real life" situation? Will students be encouraged to make sense of the situation based on extensions of their own personal knowledge and experiences? Will students' ideas be taken seriously, or will students be forced to conform to the teacher's (or author's) notion of the (only) "correct" way to think about the problem situation?

The Model Construction Principle:

Does the task create the need for a model to be constructed, or modified, or extended, or refined? Does the task involve constructing, explaining, manipulating, predicting, or controlling a structurally significant system? Is attention focused on underlying patterns and regularities rather than on surface-level characteristics?

The Self-Evaluation Principle:

Are the criteria clear for assessing the usefulness of alternative responses? Will students be able to judge for themselves when their responses are good enough? For what purposes are the results needed? By whom? When?

The Model-Documentation Principle:

Will the response require students to explicitly reveal how they are thinking about the situation (givens, goals, possible solution paths)? What kind of system (mathematical objects, relations, operations, patterns, regularities) are they thinking about?

The Model Generalization Principle:

Does the model that is constructed apply to only a particular situation, or can it be applied to a broader range of situations?

The Simple Prototype Principle:

Is the situation as simple as possible, while still creating the need for a significant model? Will the solution provide a useful prototype (or metaphor) for interpreting a variety of other structurally similar situations?

In the section that follows, the problem activity will be presented. This problem was designed to relate to similar employment experiences the students may have had. Next, the corresponding student results, along with our interpretations, will provide evidence of the models and modeling cycles which occurred as a particular group of students solved the problem. In each interpretation we will illustrate the meaning of the particular model and its function in the modeling cycle.

THE PROBLEM ACTIVITY

The activity that follows is part of the PACKETS program for Middle School Mathematics, developed by the Educational Testing Service for the purpose of portfolio assessment, according to the principles described above. This problem was based on a context that was described in a "math-rich" newspaper article that was discussed by the class as a whole on the day before the "Making Money" problem was presented.

Making Money

Last summer Maya started a concession business at Wild Days Amusement Park. Her vendors carry popcorn and drinks around the park, selling wherever they can find customers. Maya needs your help deciding which workers to rehire next summer.

Last year Maya had nine vendors. This summer, she can have only six—three full-time and three half-time. She wants to rehire the vendors who will make the most money for her. But she doesn't know how to compare them because they worked different numbers of hours. Also, when they worked makes a big difference. After all, it is easier to sell more on a crowded Friday night than on a rainy afternoon.

Maya reviewed her records from last year. For each vendor, she totaled the number of hours worked and the money collected—when business in the park was busy (high attendance), steady, and slow (low attendance). (See the table.) Please evaluate how well the different vendors did last year for the business and decide which three she should rehire full-time and which three she should rehire half-time.

Write a letter to Maya giving your results. In your letter describe how you evaluated the vendors. Give details so Maya can check your work, and give a clear explanation so she can decide whether your method is a good one for her to use.

HOURS WORKED LAST SUMMER

	JUNE			*JULY*			*AUGUST*		
	Busy	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*
MARIA	12.5	15	9	10	14	17.5	12.5	33.5	35
KIM	5.5	22	15.5	53.5	40	15.5	50	14	23.5
TERRY	12	17	14.5	20	25	21.5	19.5	20.5	24.5

JOSE	19.5	30.5	34	20	31	14	22	19.5	36
CHAD	19.5	26	0	36	15.5	27	30	24	4.5
CHERI	13	4.5	12	33.5	37.5	6.5	16	24	16.5
ROBIN	26.5	43.5	27	67	26	3	41.5	58	5.5
TONY	7.5	16	25	16	45.5	51	7.5	42	84
WILLY	0	3	4.5	38	17.5	39	37	22	12

MONEY COLLECTED LAST SUMMER (IN DOLLARS)

	JUNE			***JULY***			***AUGUST***		
	Busy	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*
MARIA	690	780	452	699	758	835	788	1732	1462
KIM	474	874	406	4612	2032	477	4500	834	712
TERRY	1047	667	284	1389	804	450	1062	806	491
JOSE	1263	1188	765	1584	1668	449	1822	1276	1358
CHAD	1264	1172	0	2477	681	548	1923	1130	89
CHERI	1115	278	574	2972	2399	231	1322	1594	577
ROBIN	2253	1702	610	4470	993	75	2754	2327	87
TONY	550	903	928	1296	2360	2610	615	2184	2518
WILLY	0	125	64	3073	767	768	3005	1253	253

Figures are given for times when park attendance was high (busy), medium (steady), and low (slow).

Student responses and corresponding interpretations

The student responses contained in this section come from a seventh grade "average ability" inner-city classroom. The students worked in three-person teams, with the members being assigned by the teacher. This particular teacher placed an emphasis on portfolio-based assessment; therefore, these students had considerable prior experience working on at least ten projects similar in size to the "Making Money" problem.

For this activity, the students worked at small tables where a "tool kit" was available that included three graphing calculators and other standard classroom tools. The "work station" also included a Macintosh computer with a 12" color monitor, and software for word processing, spreadsheets, drawing, and making geometry constructions. The teacher passed out the problem and told the students that they were to complete their letter describing a procedure for deciding who to hire by the end of the next day's class.

The solution process that follows includes significant segments from a transcript for a group of students whose names were Alan, Barb, and Carla. Most of the graphs that are shown were originally produced using graphing calculators. But, when the teams presented their work in class, they used posters that contained re-drawn versions of their favorite graphs; and, these graphs usually were constructed using a computer-based graphing spreadsheet and a color printer.

{Approximately 5 minutes pass as students read the problem & discuss it.}

Alan: Oh God. We've gotta add up all this stuff. … You got a calculator?

Barb: They're in here {the toolbox}. … Here. {she finds two TI-83 calculators in the toolboxes}.

Approximately five more minutes pass while Alan, Barb, and Carla add the numbers in various rows or columns of the table. Since the three students had made no effort to coordinate their efforts, each went off in a slightly different direction. For example, Barb and Carla both added numbers in the first row of the table (which shows the number of hours that Maria worked); whereas Alan added the numbers in the first column (which shows the number of hours that all students worked during the busy periods in June).

Carla: (looking at Barb) What'd you get? … I got 159.
Barb: Yep. … um … That's what I got.
Alan: I got, let's see, … 116.
Barb: You punched them in wrong. … Here, you read them {the numbers} and I'll punch 'em in.
Alan: (pointing to the numbers in the table) 12.5, 5.5, 12, 19.9, 19.5 …

Interpretation #1: Inconsistent use of a "hodge podge" of several unstated and uncoordinated ways of thinking

This team's first interpretation of the problem was similar to those generated by most other groups. That is, when students first began to work on the problem:

1) They tended to worry most about "What should I do?" rather than "What does this information mean?" Therefore, their first interpretations focused on computation, and the information that was given was treated as though no data interpretation or mathematization was necessary. Also, when computation was done, it nearly always involved only two-item combinations; it did not involve computations of whole rows or whole columns of numbers.

2) They tended to focus on only a small subset of the information, and they tended to focus on isolated pieces of information rather than focusing on underlying patterns and regularities. For example, Alan, Barb, and Carla focused on only the first information that impressed them most. That is, they focused on only the rows or columns in the table that showed the number of hours that each worker worked. This emphasis was not based on a thoughtful selection about which information was most important. It was simply the first information that came to their attention.

3) Their early interpretations seldom consisted of a single coherent way of thinking about givens, goals, and possible solution procedures; instead, they tended to involve a hodge podge of several unarticulated and undifferentiated points of view. That is, different students think in different ways; and even for a given individual, they sometimes switch (without noticing) from one way of looking at the problem to another way. For example, in the transcript that is given here, when Alan finished adding the first column of numbers in the top half of the table, he began to add the first column of numbers in the bottom half; there was no evidence that he noticed that the top half of the table dealt with hours worked and that the bottom half dealt with money earned. In fact, later in the

session, Alan tried to subtract data in the top table from data in the bottom table (he tries to subtract hours for dollars, e.g., $690 - 12.5 hours = ?).

4) They tended to focus only on numbers, and ignored quantity types. For example, the quantity “12.5 hours” usually was read as “twelve point five.” This emphasizes “how much” but ignores “of what.”

Next, Alan, Barb, and Carla spent approximately five minutes calculating the total amount of time that other workers worked. Carla recorded results in the last column of her table. The table of sums that they produced corresponds to the graph shown below.

Interpretation #2: Focusing on total number of hours for each worker

The graph and table shown here focused on only the total number of hours that each worker worked. In presentations of their results, the notions of “seniority” or “willingness to work” were common justifications that students used for emphasizing “hours worked.”

Unlike many other groups that produced the preceding graph as part of their final presentations, Alan, Barb, and Carla did not bother to produce the graph shown below. They only produced the table of sums that would have led to this graph. This seemed to be true for several reasons. First, the table of sums that Alan, Barb, and Carla produced was, in itself, enough to enable them to go on to a new and improved way of thinking about the information that was given. Second, at this point in the session, Alan, Barb, and Carla were only using their calculators to operate on pairs of numbers; they were not operating on whole lists of numbers. Therefore, they were not entering data into their calculators (or their computer) in a form that made it easy for them to produce automatic graphs.

MARIA	159
KIM	239.5
TERRY	174.5
JOSE	226.5
CHAD	182.5
CHERI	163.5
ROBIN	298
TONY	294.5
WILLY	173

Alan: OK, so who should we hire? {Alan was looking at Carla’s table of sums.}

Barb: Robin looks good. … {pause} So does Tony.

Alan: Maybe Kim. {pause}

Carla: Hey! We ought to look at money, not hours. … Money is down here {pointing to the second half of the table which shows the amount of money each student earned}.

Alan: Yep, money. {Approximately 1 minute passes, as students think and look at the table.}

Barb: OK, let's add these. (pointing to the rows in the second half of the table).

Barb: Here, you do Maria. (gesturing to Alan) You do Kim. (gesturing to Carla) And, I'll do Terry.

Alan, Barb, and Carla divided the task into several different tasks, with each working on different parts. In this way, more planning, monitoring, cross-checking, and rethinking tended to occur. Alan, who seemed to be insecure about using a calculator, begins to act as a facilitator and as a monitor for the group, rather than as a person who is actually doing the calculations.

Next, approximately three minutes pass as the students calculate sums in the second half of the table. At this time, Barb becomes the temporary recorder for the group. She takes several minutes to collect the results from the group, and to record these sums in a column (like the one that Carla had constructed earlier).

Interpretation #3: Focusing on the total number of dollars that each worker earned: Some teams essentially quit working on the problem at this point. For these groups, their presentations often included a graph like the one shown below. … Again, probably for the same kinds of reasons as for interpretation #2, Alan, Barb, and Carla used only a table of sums; they did not bother to construct the graph shown below.	MARIA KIM TERRY JOSE CHAD CHERI ROBIN TONY WILLY	\$8,196 \$14,921 \$7,000 \$11,373 \$9,284 \$11,062 \$15,271 \$13,964 \$9,308

Alan: So, who's the best? … {pause} … Robin's best. She got "fifteen two seventy-one." … And, Kim got "fourteen nine twenty-one." Who's next?

Carla: Tony. … He got "thirteen nine sixty-four."

Barb: This isn't fair. Some guys got to work a lot more than others. … Look at Robin and Tony. They worked more than everybody else. That's why they made more money. If Maria worked that much, she'd have made that much money too.

{Mumbling. More than 2 minutes pass.}

At this point in the session, nobody picks up on Barb's suggestion to investigate the relationship between "dollars earned" and "hours worked." Nonetheless, later in the session, Barb comes back to this same suggestion, and at that time, it leads to the idea of investigating "dollars-per-hour" for each worker. Now, however, the students investigate changes in the dollars earned across time.

Barb: Look, Willy didn't work at all in June {pointing to the zeros by Willy's name in the original table.} But, he was doing great in August {pointing to the \$3005 by Willy's name in the August column of the original table.} … Let's just see how much everybody got, totally, in August.

Next, the group spent approximately 10 minutes making a table showing the total number of dollars each worker earned each month. At first, the group only made a list of the totals for August; but, when they were finished with August, they made a table showing all three months. Also, after the values were calculated using calculators, Carla entered the results into the computer spreadsheet. [note: Alan, Barb, and Carla never used the spreadsheet to calculate values; they only used it to record information and to graph results.]

Interpretation #4a: Using a table to focus on the total number of dollars each month.

It is noteworthy that the preceding table was put together in a top-down fashion. Earlier tables were simple lists, and even these lists were created by doing the individual calculations first, and then organizing these results into a well-organized form. The organizational system was not generated first and used as a form to guide the computations that were performed. That is, each of the earlier lists were constructed in a bottom-up fashion.

Dollars Earned Each Month

	June	July	August
MARIA	$1922	$2292	$3982
KIM	$1754	$7121	$6046
TERRY	$1998	$2643	$2359
JOSE	$3216	$3701	$4456
CHAD	$2436	$3706	$3142
CHERI	$1967	$5602	$3493
ROBIN	$4565	$5538	$5168
TONY	$2381	$6266	$5317
WILLY	$189	$4608	$4511

Alan: Look at old Willy. He's really catching on {at the end of the summer}. … Look, back here {in June} he only made a hundred and eighty-nine bucks; but, out here {in August} he was really humming.

Barb: I think August should count most. Then July. … I don't think June should count much. They were just learning.

Alan: How are we going to do that.

Barb: I don't know. Just look at them {the numbers in the table} I guess. {pause}

Barb: Let's see, out here {in August} Kim was best. … Then Robin, no Tony. … Then Robin. … I think they're the top three. Kim, Robin, and Tony. … How'd they do in July?

Barb: Wow! Look at Kim. She's still the best. … But, uh oh, look at Cheri. She was real good in July.

Alan: Let's line 'em up in July. Who's first.

Barb: Kim. … {pause} Then Tony, and Cheri, and Robin. … {long pause} … Then Willy, Chad, and Jose. … {long pause} … And, these guys weren't very good {referring to Maria, Terry}.

While Barb was doing most of the talking and overt work, Alan was watching and listening closely. But, Carla was off on her own playing with the computer's spreadsheet, and entering lists of numbers. At this point, Carla re-enters the conversation.

Carla: Look you guys, I can make a graph of this stuff. Look.

For the next four minutes, Carla used the computer to flip back and forth, showing the three graphs that she had made, explaining how she made the graphs, and pointing out who was the top money earner each month.

Interpretation #4b: Using a graph to focus on the total number of dollars earned each month Similar graphs were made for July and August.	

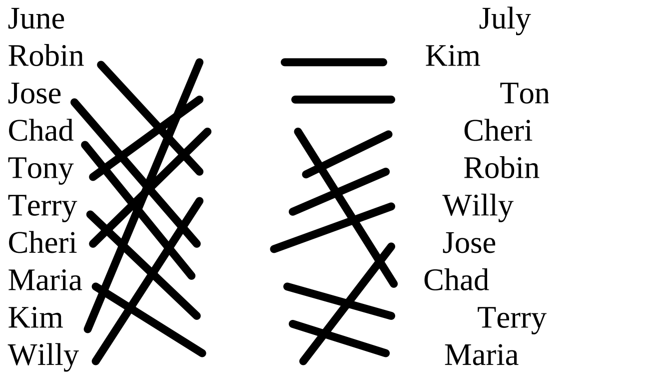
Barb: OK, let's, like, line 'em up for each month.
Alan: You started doing that.
Barb: OK, you {Alan} read 'em off and I'll write 'em down.

For approximately five minutes, Alan, Barb, and Carla worked together to get a list of "top money makers" each month.

Alan: Look, Kim was top in July and August; and, so was Tony. … Robin was next in August; but, she wasn't as good in July. … {pause} … But, she {Robin} was really good in June. … {pause} … I think August is most important because some of them were just learning. … August is how they'll probably do next summer.

Interpretation #5: Focusing on trends in rank across time.

The students noticed that the rankings were somewhat different each month; so, the “trends” shown here were used as an early attempt to reduce this information to a single list.



Approximately five more minutes passed while each of the three students nominated workers that they believed should be hired, based on rankings and trends in the preceding table. In most cases, when students spoke in favor of a given worker, they made up some sort of “cover story” to account for the “ups” and “downs” in the performance of the worker. These “cover stories” involved the following kinds of possibilities: (1) some workers learned and improved, while others got bored; (2) some weren’t able to work as much as others; (3) some were good during busy periods, but not during slow periods. In these discussions, the students started to pay attention to the fact that the months might not be equally important (e.g., July is the busiest month, August might be the best indicator of current abilities), and that busy, steady, and slow periods might not be equally important (e.g., part-time workers wouldn’t be hired during slow periods). In addition, the students began to express concerns about the fact that they would like to have had some additional information that was not available (Who really needed a job badly? Who was willing to work when they were called?). Finally, as Carla is looking at the three-column chart that showed trends (see interpretation #5), she got the idea to make a similar graph using the computer; and this idea leads to interpretation #6.

Carla: I can make a graph like that {pointing to the table that was used in interpretation #5} with the computer. Want’a see? (see Interpretation #6)
Alan: Wow! Neat! How’d you do that?
{Carla explains again how she made the graphs using the computer.}
Alan: Now who do we pick. … Who’s this?
Carla: Um, let’s see, it’s Kim. … And, this is … um … Tony.
Alan: Who’s this?
Barb: Let me see.
Carla: Oh, it’s Robin.
Barb: So, we’ve got Kim … Tony, and Robin. Who’s next? {pause}
Carla: What about this guy? … Who is he? … Um, it’s Cheri. … Look, she was really good here. But, then she screwed up.

Interpretation #6: Focusing on trends in money earned for June, July, and August.

Carla's graph was a line graph showing the total number of dollars that each worker earned for June, July, and August.

Trends in Money Earned for June, July, & August

Barb: How we gonna decide which of these guys to hire? They were all good some and bad some. … {long pause} … How many were we supposed to hire anyway? … {pause} … Look at the problem {speaking to Alan}. What does it say?

… {long pause} …

Alan: We're supposed to hire three full time and three part time.

… {long pause} …

Alan: I think we should hire Willy. He was good here {pointing to July and August} … and he didn't get to work much here {pointing to June}.

Interpretation #7: Using telescoping decision rules.

Up until this point in the session, the students implicitly seemed to assume that the best way to choose workers should be to use a single rule for ranking the workers. Then, if this list was successful in ranking workers from "best" to "worst," the top three workers could be hired for full time, and the next three workers could be hired for part time. Unfortunately, no single rule seemed to work to form a single list. For example, both Barb and Carla suggested the idea of using some sort of average. But, this idea was not considered in detail, because the type of averages that were mentioned didn't seem to involve equally important quantities. Therefore, the students began to consider more sophisticated decision-making rules. For example, one rule involved the following kind of two-step process. First-round decisions about who to hire could be based on the ranking in August alone; then, second-round decisions could be based on the ranking in July alone (or based on busy periods alone).

Barb: Look you guys. Some of these people got to work a lot more than others. … That's not fair. Look, Willy didn't get to work at all back here {in June}.

Carla: So, what're we gonna do?

{Mumbling. More than 1 minute passes.}

Alan: Here. I'm trying something. … I'm subtracting how much each guy worked. That'll kind of even things out. … I worked for a guy who did that once. We were cleaning up trash and he wanted us to work fast.

Interpretation #8: Subtracting time scores from money scores

The most important characteristic of this new idea is that, for the first time, it took into account a relationship between the amount of money that was earned and the amount of time that was spent working. But, because the numbers in the tables didn't include any

unit labels, nobody noticed that it might not make sense to subtract hours from dollars. Nonetheless, neither Barb nor Carla were convinced that the idea made sense. … What did make sense to Barb and Carla was to apply lessons they had learned from their own prior "real life" experiences to help them make decisions in case of the "summer jobs" problem. Therefore, the team didn't pursue Alan's suggestion. Instead, Alan's suggestion was used as a (transitional) way of thinking which led to a better idea which Barb suggested that would take into account *both* time and money.

Barb: Hey, that's a good idea! We could figure out dollars-per-hour. … I did that for my jobs last summer.

Interpretation #9: Focusing on Dollars-per-hour Barb wasn't really paying close attention to Alan's idea. The new ideas that she heard were to think about the situation in the same way that she thought about her own past jobs. That is, both Alan and Barb were using past "real life" experience to make sense of the current problem. Therefore, Barb thought in terms of dollars-per-hour.

For the remaining minutes of the class, Alan, Barb, and Carla went back to the original data tables and started calculating dollars-per-hour. … As class ended, they decided that, to prepare for the next day's class, each student should bring a graph showing dollars-per-hour for the workers. Then they planned to use these graphs to make final decisions about who to hire. The graphs on the following page show what each student brought to class the next day.

Interpretation #10a: Alan's dollars-per hour graph based on sums for the whole summer First, Alan calculated the total amount of money that each worker earned for the whole summer. Then, he calculated how much time they worked altogether. Finally, for each worker, he divided total dollars by total time.	

Interpretation #10b: Barb's dollar-	Total Dollars-per-Hour Each Month

per-hour graph based on sums for each month First, Barb calculated the total amount of money that each worker earned for each month. Then, she calculated how much time they worked each month. Finally, for each month, she divided dollars earned by time worked.	

Interpretation #10c: Carla's graph showing the average dollars-per-hour each month (where the average is taken across busy, steady, and slow periods).

	June			**July**			**August**		
	Busy	Steady	Slow	Busy	Steady	Slow	Busy	Steady	Slow
Maria	$55.20	$52.00	$50.22	$69.90	$54.14	$47.71	$63.04	$51.70	$41.77
Kim	$86.18	$39.73	$26.19	$86.21	$50.80	$30.77	$90.00	$59.57	$30.30
Terry	$87.25	$39.24	$19.59	$69.45	$32.16	$20.93	$54.46	$39.32	$20.04
Jose	$64.77	$38.95	$22.50	$79.20	$53.81	$32.07	$82.82	$65.44	$37.72
Chad	$64.82	$45.08		$68.81	$43.94	$20.30	$64.10	$47.08	$19.78
Cheri	$85.77	$61.78	$47.83	$88.72	$63.97	$35.54	$82.63	$66.42	$34.97
Robin	$85.02	$39.13	$22.59	$66.72	$38.19	$25.00	$66.36	$40.12	$15.82
Tony	$73.33	$56.44	$37.12	$81.00	$51.87	$51.18	$82.00	$52.00	$29.98
Willy		$41.67	$14.22	$80.87	$43.83	$19.69	$81.22	$56.95	$21.08

Interpretation #10c, continued Note: Carla got some help from her brother, who apparently suggested	Average Dollars-per-Hour Each Month (Across Busy, Steady, & Slow Periods)
the idea of an average. First, Carla calculated the dollar-per-hour for each cell in the matrix shown. Then, for each month, she calculated the average of the rates for the busy, steady, and slow periods. This procedure assumes (incorrectly) that the students intended to treat busy, steady, and slow periods as being equally important!	

For approximately the first twenty minutes of the second class, Alan, Barb, and Carla showed one another their rate-per-hour graphs, and they explained how the graphs were made. Then, for each graph, the team as a whole worked to try to decide which workers should fall into the categories: full time, part time, and don't hire. For Alan's list, the ranking was easy to read directly from the graph that he had drawn. But, for Barb's graph and for Carla's graph, it was not as obvious to determine which workers ranked first, second, third, and so on. Therefore, for both of these graphs, the teams used telescoping methods of decision making. That is, first-round (tentative) decisions were based on performances in August alone. Then, to make decisions about difficult cases, information was used from July (or from June). The results are shown below.

Interpretation #11: Three different lists were generated that ranked workers from lowest to highest based on the dollar-per-hour graphs that the students had produced.

	Alan's List	**Barb's List**	**Carla's List**
	Cheri	Kim	Cheri
FULL TIME	Kim	Cheri	Jose
	Willy	Willy	Kim
	Maria	Jose	Tony
PART TIME	Robin	Chad	Maria
	Chad	Robin	Willy
	Jose	Maria	Chad
DON'T HIRE	Tony	Tony	Robin
	Terry	Terry	Terry

Because the preceding three lists were somewhat different, Alan, Barb, and Carla tried to make a new list (which they called their "agreement list") showing points of agreement among the three lists. While the students are discussing the possibilities, Carla comes up with a new idea.

Carla: Look, on my list, Cheri, Jose, and Kim all got A's. … Tony, Robin, and Willy got B's. And, Chad, Robin, and Terry got C's. … What did they get on your lists?

Alan: What do you mean?

Carla: Give me your list, I'll show you. … {pause} … See. Cheri got an A, and so did Kim and Willy.

Barb: What are you guys doing?

Carla: Here watch.

For approximately the next five minutes, Carla asks the other two students to give her information to fill in the "grading scale" shown on the following page.

Interpretation #12: Generating a "grading scheme."

For each list, a "grading scheme" is imposed that is similar to those used for tests in class. The scores are then combined (treating each of the rankings as if they were independent ratings).

	Alan's List	Barb's List	Carla's List		Combined
Cheri	A	A	A		A
Kim	A	A	A		A
Willy	A	A	B		A-
Jose	C	B	A		B
Robin	B	B	C		B-
Chad	B	B	C		B-
Maria	B	C	B		B-
Tony	C	C	B		C+
Terry	C	C	C		C

Alan: So, it looks like the full time people should be Cheri, and Kim, and Willy. … And part time should be Jose, and … uh oh! Who should we pick next? Maria, Robin, or Chad?

Barb: Yeah. Tony and Terry are out.

Alan: These other guys are pretty close. … It's not fair to just pick one.

Carla: Maybe one of these guys really needs a job. I'd think we should hire guys who really need a job. Maybe Willy doesn't really need a job. Maybe Jose really needs one.

Alan: Some of these guys probably didn't get to work at the good times. {pause}

Barb: Let's make more graphs like these {pointing to her rate-per-hour graphs in interpretation #10C} for the slow times, and the steady times, and the fast times.

More than 12 minutes pass while Alan, Barb, and Carla worked together to make graphs comparing dollars-per hour for busy, steady, and slow periods.

Interpretation #13: A telescoping series of rules. First round decisions are based on interpretation #12. Then, second round decisions are made by comparing dollars-per-hour for busy, steady, and slow periods. (The graph for busy periods is shown, similar graphs were made for steady and slow periods.)	

Barb: {looking at the preceding graphs} I don't think this helps much.

Carla: {looking at the preceding graphs} So, which one should we hire? Maria, Robin, or Chad?

Alan: Look, Maria's only best during slow times. But, we don't really care about slow times. We're only going to hire part time people when things are happening … fast times. {pause}

Barb: Wait a minute. Maria's not so bad. Look, um, she's better that Robin during steady times. … and Chad too.

Approximately eight minutes pass in which Alan, Barb, and Carla looked back over the graphs that they brought to class, and the work they did earlier in the period. In these discussions, they offer "stories" that might possibly explain patterns in the dollars-per-hour for various workers. In the end, they reached an agreement on the following points:

1) slow periods should not be treated as being very important, because (a) most of the money would be made during busy or steady periods, and (b) part time workers would not be hired during slow periods.

2) performance in August (and, to a lesser extent, July) should be treated as being most important, because (a) it took into account learning and improvement, and (b) it was the most recent indicator of worker capabilities.

Carla: We've got to write up our report. … What should we do?

Barb: I think we should make another graph like the one I made before {i.e. Interpretation #10b} … only this time leave out slow times.

Carla: OK, you do that. … I'll get the poster board and stuff.

For the remainder of the class period, Alan, Barb, and Carla worked together to produce a large poster like the one shown on the following page.

Interpretation #14: A telescoping series of rules based on dollar-per-hour trends for (only) busy and steady periods.

Dear Maya,
We think you should hire Kim and Cheri and Jose for full time, and we think you should hire Willy and Chad and Tony for part time. Look at this graph to see why these people are best.
The graph is only about busy times and steady times. You don't make much money during slow times, and you won't hire people for slow times.
Some workers got better at the end of the summer. But, some didn't get better. So, August is most important, and July is also important. July is when you make the most money.

Alan, Barb, and Carla

IMPLICATIONS

The interpretations provided for the responses given by Alan, Barb, and Carla were intended to help the reader gain insight into the various “models” and “modeling cycles” that were used during the solution process. Notice that the students cycled through models that began with informal intuitions (performing simple computations without any interpretation or mathematization of the data) and proceeded toward more formal systems (looking at trends, averages, and graphical analysis and representations of data). Furthermore, the students began to go beyond thinking *with* conceptual models to also thinking *about* them. That is, they analyzed the underlying assumptions, strengths, and weaknesses associated with each model. For example, they could have stopped after calculating “hours worked” as in interpretation #2, but Carla pointed out that they should look at money earned before making a decision. Or, they could have decided who to hire based upon total number of dollars earned (as in interpretation #3), but Barb pointed out that “some guys got to work a lot more than others.” Each revision made during the solution process involved related, but qualitatively distinct models, and these were needed to produce the increasingly useful solutions to this problem. If the students were not given sufficient time, resources, and opportunities to cycle through the different models, powerful ideas may never have been developed and applied.

It is useful to note that the teacher was not the one who prompted the students to use the different conceptual models. In fact, she did not intercede or interfere with the flow of events or ideas. She did however, create an environment in which students could discuss, defend, and justify their own solutions. In addition, she provided the students with access to realistic tools and resources such as calculators and computers.

What prompted the students to develop and use the powerful conceptual models discussed in the previous section? Consider the characteristics of “model-eliciting” activities stated previously. First, the problem focused on a “real” issue which required a decision that needed to be addressed (not just a school-math question related to it). The students knew who was asking for the information (Maya), why she was asking (to help her decide who to rehire), and the criteria that would ultimately influence the quality of the response. Next, the students needed to justify and explain their decisions by describing underlying assumptions and conditions, and they had to analyze and assess alternative conclusions, explanations and interpretations generated by themselves and each other. Third, the statement of the problem required them to explicitly reveal how they were thinking about the situation (the givens, goals, possible solution paths, etc.—recall that Maya asked them to provide a clear explanation to help her decide whether the method chosen for evaluating the vendors is a good one for her to use). Fourth, the students had to make judgments for themselves about issues such as whether (or in which directions) current solutions needed to improve. Fifth, the problem prompted the students to consider models which ultimately can be used to generate answers to a whole class of questions in a whole range of situations (models for dealing with trends, averages, data analysis, etc.). This is in sharp contrast to textbook problems which generally ask students to produce nothing more than a specific response to a particular question, and not produce a model at all. Last, the problem situation was designed to be as simple as possible while still creating the need for a significant model. In total, these characteristics pushed students to construct, manipulate, extend, and refine powerful mathematical models.

The interpretation boxes were intended to highlight the statistical uses that emerged as the students solved the problem activity. These included: multiple views of data, both graphical and tabular; measures of central tendency; analyses of trends; and procedures for combining data. Since mathematical knowledge and abilities develop along a number of dimensions such as from concrete to abstract, from specific to general, from global/undifferentiated to refined, or from intuitions to formalizations, it is important for teachers to use model-eliciting activities at the beginning of instruction to gather valuable assessment information. As students work on these activities, they not only reveal the concrete/intuitive/informal understandings (and misunderstandings) that they have, but they can also extend, refine, or integrate these ideas to develop new levels of understandings. One of the points to be made with regard to assessment, is that when teachers observe students solving model-eliciting problem activities, the information that they get is similar to the kind that might have resulted from one-to-one clinical interviews. Therefore, teachers learn about students' strengths and weaknesses so that: (1) they can avoid re-teaching ideas that students already know; (2) they can focus on the key issues that students do not understand; and (3) they can use existing understandings and capabilities as foundations for new knowledge and abilities. Model-enhancing activities can be used as part of a student's portfolio (see Chapter 13). Note however, they differ quite significantly from multiple-choice methods (see Chapter 16).

In sum, this chapter was intended to show that when students are provided with opportunities to solve model-eliciting activities in which they can assess and monitor their own work using realistic tools, and when more options are available concerning modes of responses and solution paths, students can construct, modify, and refine powerful conceptual models for dealing with data analysis and statistics. We believe that model-eliciting tasks are important in statistics education because they offer a window into statistical reasoning processes or conceptual structures in statistics education that otherwise may be difficult or impossible to assess using traditional methods.