

AN EXAMINATION OF THE LEVELS OF COGITIVE DEMAND REQUIRED BY PROBABILITY TASKS IN MIDDLE GRADES MATHEMATICS TEXTBOOKS

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ABSTRACT

We analyze probability content within middle grades (6, 7, and 8) mathematics textbooks from a historical perspective. Two series, one popular and the other alternative, from four recent eras of mathematics education (New Math, Back to Basics, Problem Solving, and Standards) were analyzed using the Mathematical Tasks Framework (Stein, Smith, Henningsen, & Silver, 2000). Standards-era textbook series devoted significantly more attention to probability than other series; more than half of all tasks analyzed were located in Standards-era textbooks. More than 85% of tasks for six series required low levels of cognitive demand, whereas the majority of tasks in the alternative series from the Standards era required high levels of cognitive demand. Recommendations for future research are offered.

Keywords: *Probability; Curriculum; Mathematics textbook content analysis; Mathematical tasks; Cognitive demands; Middle grades mathematics*

1. INTRODUCTION

1.1. THE EMERGENCE OF PROBABILITY IN SCHOOL MATHEMATICS

Consumers and citizens in today's information-rich society need to have an understanding of probability. Shaughnessy (1992) stated, "There is perhaps no other branch of the mathematical sciences that is as important for *all* students, college bound or not, as probability and statistics" (p. 466, emphasis in original). Despite the importance of probability and statistics, many children and adults hold misconceptions about probability (Garfield & Ahlgren, 1988; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993). In fact, Garfield and Ahlgren (1988) stated that "inappropriate reasoning [in probability and statistics] is...widespread and persistent...and similar at all age levels" (p. 52).

Instruction in probability should provide experiences in which students are allowed to confront their misconceptions and develop understandings based on mathematical reasoning (Garfield & Ahlgren, 1988; Konold, 1983, 1989; Shaughnessy, 2003). Due to the widespread nature of probabilistic misconceptions among adults, such instruction may not have occurred for all students in recent decades. Perhaps probability topics were not present in textbooks, or perhaps these topics were present in textbooks but omitted from instruction (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Shaughnessy, 1992).

Teachers may omit topics for a number of reasons, including a lack of preparation to teach topics from probability due to their own lack of experience or misconceptions (Conference Board of the Mathematical Sciences, 2001), or a subsequent lack of confidence in their ability to teach such topics. In some cases, a teacher's interpretation of and orientation to the curriculum may constrain how what is printed in the textbook is communicated to the class (Remillard & Bryans, 2004). Moreover, teachers' interpretations of the textbook may be in opposition to the intentions of the authors (Lloyd, 1999; Lloyd & Behm, 2005). In such cases, a teacher may use all of the probability tasks in an investigation-oriented textbook, but present these tasks in a traditional manner by providing students with explicit rules, formulas, and repetitive practice problems. Alternatively, teachers may not have time to teach all of the material present in the textbook, and omit probability lessons simply for a lack of sufficient time. If probability topics are taught, the lessons as presented in textbooks may not sufficiently address students' misconceptions.

Over the past several decades, probability has emerged as an important topic for all students to learn, particularly those in the middle grades. Due to the growing emphasis on the topic of probability in recommendations from professional organizations, one might reasonably expect to observe changes in textbooks. Unfortunately, there have not been any systematic examinations of the composition of textbooks as they have evolved over time, particularly in relation to probability.

1.2. TEXTBOOK USE AND LEVEL OF COGNITIVE DEMAND

Textbooks are common elements in classrooms throughout the world, and are ubiquitous in mathematics classrooms in the United States. Textbooks are present not only in classrooms, they are also frequently used by teachers and students, and influence the instructional decisions that teachers make on a daily basis (Robitaille & Travers, 1992; Tyson-Bernstein & Woodward, 1991). Recent studies have revealed that most middle-grades (grades 6-8) mathematics teachers use most of the textbook most of the time (Grouws & Smith, 2000; Weiss, Banilower, McMahon, & Smith, 2001). Grouws and Smith (2000) observed that the mathematics teachers of three fourths of the eighth grade students involved in the 1996 National Assessment of Educational Progress (NAEP) reported using their textbook on a daily basis. Weiss et al. (2001) found that two thirds of middle-grades mathematics teachers "cover" at least three fourths of the textbook each year. These findings tend to agree with results of research on students' use of mathematics textbooks as well. In the 2000 administration of the NAEP, 72% of participating eighth graders stated that they did mathematics problems from a textbook every day (Braswell et al., 2001).

After analyzing the levels of cognitive demand of mathematical tasks, QUASAR [Quantitative Understanding: Amplifying Student Achievement and Reasoning] project researchers noted that students "need opportunities on a regular basis to engage with tasks that lead to deeper, more generative understandings about the nature of mathematical concepts, processes, and relationships" (Stein, Smith, Henningsen, & Silver, 2000, p. 15). They also found that teachers implementing tasks with high levels of cognitive demand rarely selected tasks from commercial textbook series (Stein, Grover, & Henningsen, 1996).

2. RESEARCH OBJECTIVES

Because textbooks have a marked influence on what is taught in mathematics classrooms, it is important to investigate curricular materials that many mathematics teachers use and the potential of such resources to impact students' opportunities to learn probability. Accordingly, the research reported in this article addressed the following research questions: What is the nature of the treatment of probability topics in middle grades mathematics textbooks? How has the nature of the treatment of probability changed over the past 50 years and across popular textbooks series and alternative (or innovative) textbook series? More specifically, what levels of cognitive demand are required by tasks and activities related to probability, and what are the trends in the required level of cognitive demand over the past 50 years?

Heretofore there has not been any systematic review of the content of textbooks over time. Thus, our study is intended to highlight any differences that have come about through different eras of mathematics education, and how these differences coincide with the contemporary recommendations for the inclusion of probability in the school mathematics curriculum. Moreover, it is our goal to reveal the degree to which textbooks have maintained the status quo in terms of the content and level of cognitive demand required by tasks.

3. THEORETICAL CONSIDERATIONS

3.1. RECENT TEXTBOOK CONTENT ANALYSES

In a pivotal study, Project 2061 (American Association for the Advancement of Science [AAAS], 2000) analyzed thirteen contemporary mathematics textbook series written specifically for middle grades students. Their sample of textbooks included four series developed with support from the National Science Foundation; the other nine series were popular textbooks from the late 1990s. They evaluated each series according to a set of benchmarks related to the core content that should be present in middle grades mathematics instruction: number concepts, number skills, geometry concepts, geometry skills, algebra graph concepts, and algebra graph skills. It is important to note that topics from probability were excluded from their analysis. The research team examined the student and teacher editions of each textbook, specifically attending to lessons that dealt with their selected benchmarks. Each series was rated as having most, partial, or minimal content according to each benchmark. The research team found that only four of the series addressed four or more benchmarks in depth, and no series sufficiently addressed all of the benchmarks. Finally, in terms of quality, none of the popular textbooks were among the best rated.

Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) also analyzed the content of the textbooks in their sample according to the characteristics of lessons. These characteristics included the primary nature of lessons (concrete and pictorial vs. textual and symbolic), components of the lesson, and student performance expectations. To measure textbook lessons along these dimensions, the researchers divided lessons into blocks, "classified according to whether they constituted narrative or graphical elements; exercise or question sets; worked examples; or activities" (p. 141). The research team analyzed these blocks according to the mathematical topics that were addressed. Results related to the treatment of probability were not reported because topics from this branch of mathematics were not present in many of the textbooks. The researchers also identified the student performance expectations for each block. This analysis revealed that

mathematics “textbooks across all populations were mostly made up of exercises and question sets” (p. 143). Additionally, over the three grade levels, the amount of narrative and worked examples increased, whereas the number of activities decreased. Furthermore, “the most common expectation for student performance was that they read and understand, recognize or recall or that they use individual mathematical notations, facts or objects. This is followed . . . by the use of routine mathematical procedures” (p. 128). In order to describe the characteristics of probability tasks in middle grades mathematics textbooks, we utilized a methodology similar to that described by Valverde et al. We identified all of the probability tasks within a textbook, and coded each task with the level of cognitive demand required by the task.

In an effort to provide a tool for comparing the intended, enacted, and assessed curricula, Porter (2006) developed two-dimensional languages to describe the content of the mathematics curriculum. This two-dimensional language can be presented in a rectangular matrix with *topics* as rows and *cognitive demands* (sometimes called *performance goals* or *performance expectations*) as columns. Topics are content distinctions such as “add whole numbers” or “point slope form of a line.” Cognitive demands distinguish memorizing; performing procedures; communicating understanding of concepts; solving non-routine problems; and conjecturing, generalizing, and proving. Our research utilizes methodology similar to that which Porter has described, in that we examined the content of textbooks in terms of topics and levels of cognitive demand.

Very recently, the National Research Council [NRC] (2004) issued a key report evaluating the evidence regarding the effectiveness of K-12 mathematics textbooks. The authors devoted an entire chapter to content analysis, and provided descriptions of the methodology and results on several recently published textbook evaluations in the form of content analyses (e.g., AAAS, 2000), as well as unpublished reports available on the world wide web (e.g., Adams et al., 2000; Clopton, McKeown, McKeown, & Clopton, 1999a, 1999b, 1999c; Robinson & Robinson, 1996). Consistent with the recommendations of the NRC, we address the depth of mathematical inquiry and reasoning of probability tasks in textbooks by rating these tasks according to their level of cognitive demand. Our analysis of the levels of cognitive demand required by tasks further provides insight into the engagement, timeliness, and support for diversity provided in each textbook. Textbooks containing tasks that predominately require lower levels of cognitive demand may not support student learning because students are rarely asked to grapple with difficult situations.

3.2. DEVELOPMENT OF THE MATHEMATICAL TASKS FRAMEWORK

Research on tasks as the primary unit of instruction and learning began in the late 1970s and early 1980s. During that time, Doyle (1983) provided the groundwork that would become influential in the work of the QUASAR research team. Doyle described students’ work in terms of academic tasks. He used this term to focus on the following:

- (a) the products students are to formulate, such as an original essay or answers to a set of test questions; (b) the operations that are to be used to generate the product, such as memorizing a list of words or classifying examples of a concept; and (c) the “givens” or resources available to students while they are generating a product, such as a model of a finished essay supplied by the teacher or a fellow student. Academic tasks, in other words, are defined by the answers students are required to produce and the routes that can be used to obtain these answers. (p. 161)

In later work, Doyle (1988) added a fourth component of academic tasks as “the importance of the task in the overall work system of the class” (p. 169). It should be noted here that Doyle considered individual questions, exercises, or problems as distinct academic tasks. He defined four general categories of academic tasks: memory tasks, procedural or routine tasks, comprehension or understanding tasks, and opinion tasks (Doyle, 1983). He argued that each of these categories varied in terms of the cognitive operations required to successfully complete tasks contained therein.

The research on academic tasks mentioned above provided a theoretical foundation for the Mathematical Tasks Framework developed by the QUASAR Project team (Smith & Stein, 1998; Stein et al., 1996; Stein & Smith, 1998; Stein et al., 2000). The Mathematical Tasks Framework represents the relationship between student learning and three phases of task implementation. In this model, tasks are first represented in curricular materials, then set up by teachers, and finally implemented by students in the classroom. This framework is particularly useful for our study, because it gives specific attention to tasks as they are present in textbooks.

This model further delineates four levels of cognitive demand for tasks: lower-level demands of Memorization and Procedures without Connections, and higher-level demands of Procedures with Connections and “Doing Mathematics.” Descriptors of each level of the framework appear in Figure 1. Stein et al. (1996) argued that it was important to examine the cognitive demand required by tasks because of their influence on student learning:

The mathematical tasks with which students become engaged determine not only what substance they learn but also how they come to think about, develop, use, and make sense of mathematics. Indeed, an important distinction that permeates research on academic tasks is the differences between tasks that engage students at a surface level and tasks that engage students at a deeper level by demanding interpretation, flexibility, the shepherding of resources, and the construction of meaning. (p. 459)

To date, the Mathematical Tasks Framework has not been used to analyze the levels of cognitive demand required by the tasks contained in a series of textbooks, let alone the probability tasks from series published over a 50-year period. Thus, in an effort to more fully describe the treatment of probability in textbooks, we made distinctions between those tasks that require students to (a) simply memorize information, (b) routinely perform algorithms without giving any attention to the meaning or development of the procedure, (c) focus on the meaning of a procedure or algorithm, and (d) explore and analyze the mathematical features of a situation.

4. METHODOLOGY

4.1. SAMPLE SELECTION

Recent Eras of Mathematics Education In order to determine historical trends in the treatment of probability in curricular materials, we selected two textbook series from each of the four most recent eras of mathematics education (Fey & Graeber, 2003; Payne, 2003): the New Math, Back to Basics, a focus on Problem Solving, and the advent of the National Council of Mathematics’ [NCTM] Standards.

The “New Math” era was so named by the contemporary popular media, as a descriptor of the innovative mathematics curricula that were being developed during this time period. Several of these curricula were developed as a response to the 1957 launch

Levels of Demands

Lower-level demands (Memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (Procedures without Connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands (Procedures with Connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (Doing Mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

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Figure 1. Characteristics of tasks at different levels of cognitive demand

of Sputnik and subsequent U.S. realization of the need for improvement in mathematics education (DeVault & Weaver, 1970; Osbourne & Crosswhite, 1970). Several facets of the New Math materials were met with intense opposition. A growing concern began to emerge from the public and elementary school teachers that students were unable to accurately compute (Payne, 2003). This growing concern blossomed into a full-fledged reactionary movement in the 1970s that focused students on the fundamentals of mathematics. For this reason, this era is referred to as “Back to Basics,” where the basics were primarily defined as computational skills (Usiskin, 1985).

After a decade of focused attention on procedures and algorithms, the NCTM (1980) published *An Agenda for Action*, calling for a focus on problem solving in mathematics classes during the 1980s. Other organizations (College Board, 1983; National Academy of Sciences and National Academy of Engineering, 1982; National Commission on Excellence in Education, 1983; National Science Foundation and Department of Education, 1980) also issued reports and recommendations for mathematics education. Usiskin (1985) summarized these recommendations as follows: “Taken as a body, reports from inside and outside mathematics education agree almost unanimously that ... emphasis should be shifted from rote manipulation to problem solving” (p. 15).

In 1989, the NCTM published *Curriculum and Evaluation Standards for School Mathematics*, calling for reform of mathematics education on a wide scale. In this document, the Council provided recommendations for mathematical content that ought to receive increased or decreased attention in the classroom and outlined important mathematical processes, such as problem solving and communication, that should be encouraged and fostered as students do mathematics. This document, along with *Professional Standards for Teaching Mathematics* (NCTM, 1991) and *Assessment Standards for School Mathematics* (NCTM, 1995) provided classroom teachers and mathematics educators with a conceptual anchor for reforming their practice. In an attempt to focus the reform of mathematics education into the new millennium, the NCTM (2000) published *Principles and Standards for School Mathematics*. This document represented further refinements of the earlier Standards documents in an integrated format, and provided more detailed narrative of the recommendations of the Council.

It is difficult to determine the precise beginning and end of these eras, and a significant event that marks the start of a new era (e.g., the publication of the *Curriculum and Evaluation Standards for School Mathematics* in 1989) does not necessarily immediately impact the textbooks that are published that year or the next. Nevertheless, we acknowledge the need to specify time frames for each era. Hereafter, we refer to the years 1957-1972 as the New Math era, 1973-1983 as the Back to Basics era, 1984-1993 as the Problem Solving era, and 1994-2004 as the Standards era. Table 1 displays the years that we designated as the terminal points of each era.

Table 1. Operational time frames for recent eras in mathematics education

Mathematics Education Era	Time Frame
New Math	1957-1972
Back to Basics	1973-1983
Problem Solving	1984-1993
Standards	1994-2004

For each era, we selected two series of mathematics textbooks: one series that was used by a relatively large proportion of middle-grade students in the United States, and one series that was different from “popular” textbooks at the time, possibly because of the

authors' desire to reform mathematics education by providing alternative curricular materials. We refer to the former type as *popular*, and the latter as *alternative*. We examined both popular and alternative textbooks from each era in an attempt to gain a broad perspective on the treatment of probability topics for that era.

Popular Textbook Selection In this study, we define a popular textbook series as the mathematics textbook series having the largest market share during a given era. Hereafter, these textbooks are referred to as *popular*. Textbook market share data are available (Weiss, 1978, 1987; Weiss et al., 2001) and were used to determine which textbook series was the most popular during the Back to Basics, Problem Solving, and Standards eras. In the absence of market share data for the New Math era, the popular textbook series was determined by a “professional consensus” of mathematics educators familiar with the middle-grades curriculum during the past 50 years and affiliated with the Center for the Study of Mathematics Curriculum.

For each era, the popular textbooks that were considered for selection must be intended for use with students in grades 6, 7, and 8. Furthermore, these textbooks should have been written for the “average-level” student, that is, neither remedial nor accelerated. For this reason, algebra textbooks (i.e., textbooks that primarily focused on algebra, and are geared toward more mathematically advanced students in the middle grades) such as *Algebra 1/2* (Saxon, 1980) or *Algebra through Applications with Probability and Statistics* (Usiskin, 1979), for example, were not considered in this study.

Data from Weiss (1978, 1987) and Weiss et al. (2001) yielded the following sample of popular textbooks (see Table 2): *Holt School Mathematics* (Nichols et al., 1974a, 1974b, 1974c) for the Back to Basics era; *Mathematics Today* (Abbott and Wells, 1985a, 1985b, 1985c) published by Harcourt Brace Jovanovich for the Problem Solving era; and *Mathematics: Applications and Connections* (Collins et al., 1998a, 1998b, 1998c) for the Standards era. For the New Math era, a majority of those comprising the “professional consensus” stated that *Modern School Mathematics* (Dolciani, Beckenbach, Wooten, Chinn, & Markert, 1967a, 1967b; Duncan, Capps, Dolciani, Quast, & Zweng, 1967) was one of the most (if not *the* most) popular textbook series for middle-grades students during the New Math era.

In this study, we examined only the student editions of each textbook, because we were primarily interested in the tasks that students may have encountered as they used the textbooks. We did not examine the teacher’s editions because students typically do not interact directly with the material within the teacher’s edition; the teacher usually mediates this interaction. Although research indicates that teachers also mediate a student’s interaction with the student’s textbook edition by lowering the cognitive demand for tasks (e.g., Arbaugh, Lannin, Jones, & Park-Rogers, 2006; Stein & Smith, 1998; Stein et al., 2000), it would be impossible (in most cases) to document the myriad of interactions between teachers and curricular materials over the past several decades. For this reason, we focus solely on the student editions of the textbook and acknowledge that our study was not designed to capture any teacher actions regarding implementation of the curricula.

Alternative Textbook Selection As with the popular textbooks, the alternative series that were considered needed to be written for the “average-level” student in grades 6-8, and algebra textbooks were not considered. Additionally, we intended to examine textbooks that were part of a comprehensive mathematics series. Thus, we did not consider materials from the Middle Grades Mathematics Project (Phillips, Lappan, Winter, & Fitzgerald, 1986) or the Quantitative Literacy Series (Newman, Obremski, &

Scheaffer, 1987) because they were originally written as supplemental units, not as a comprehensive stand-alone curriculum.

Identifying textbook series that were “alternative” (i.e., series that were possibly innovative, influential, or offered as a departure from the popular series of the time) requires assigning a value judgment to that series. Such value judgments are subjective and vary among individuals. In order to counter the subjectivity of this process, the aforementioned “professional consensus” was solicited to identify alternative middle-grades mathematics textbook series for each of the eras of concern.

Results from the professional consensus yielded the following sets of alternative textbook series for each of the specified eras, as depicted in Table 2. *Mathematics for the Elementary School: Grade 6* (School Mathematics Study Group [SMSG], 1962) and *Mathematics for Junior High School* (SMSG, 1961a, 1961b) were created with support from the National Science Foundation (NSF) during the New Math era. The SMSG materials were used in many classrooms across the United States, and had substantial impact on the content of several commercially-developed textbooks (Payne, 2003). *Real Math* (Willoughby, Bereiter, Hilton, & Rubenstein, 1981, 1985a, 1985b) published by Open Court during the Back to Basics era, was offered as an alternative to popular textbooks which focused almost exclusively on computation, as stated in an advertisement from the October 1977 issue of *Arithmetic Teacher*. Saxon Publishers offered *Math 65*, *Math 76*, and *Math 87* (Hake & Saxon, 1985, 1987, 1991) during the Problem Solving era as alternative to the popular textbooks of the time, and focused on an incremental development of skills. *Connected Mathematics Project* materials (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998a, 1998b, 1998c, 1998d, 1998e, 1998f, 1998g, 1998h, 1998i, 1998j, 1998k, 1998l, 1998m, 1998n, 1998o, 1998p, 1998q, 1998r, 1998s, 1998t, 1998u, 1998v, 1998w, 1998x) were created with the support of the NSF during the Standards era, and had the largest market share of all such middle-grades mathematics materials. The *Connected Mathematics Project* units were divided into grade levels according to the authors’ suggested order in *Getting to Know Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1996).

Table 2. Set of textbooks selected for analysis, with labels used for this study

Era	Type	Textbook Titles	Publisher
New Math	Popular	<ul style="list-style-type: none"> • <i>Modern School Mathematics: Structure and Use 6</i> • <i>Modern School Mathematics: Structure and Method 7 & 8</i> 	Houghton Mifflin
	Alternative	<ul style="list-style-type: none"> • <i>Mathematics for the Elementary School, Grade 6</i> • <i>Mathematics for Junior High School, Vols. I & II</i> 	Yale University Press
Back to Basics	Popular	<ul style="list-style-type: none"> • <i>Holt School Mathematics: Grades 6, 7, & 8</i> 	Holt, Rinehart, & Winston
	Alternative	<ul style="list-style-type: none"> • <i>Real Math: Levels 6, 7, & 8</i> 	Open Court
Problem Solving	Popular	<ul style="list-style-type: none"> • <i>Mathematics Today: Levels 6, 7, & 8</i> 	Harcourt Brace Jovanovich
	Alternative	<ul style="list-style-type: none"> • <i>Math 65: An Incremental Development</i> • <i>Math 76: An Incremental Development</i> • <i>Math 87: An Incremental Development</i> 	Saxon Publishers
Standards	Popular	<ul style="list-style-type: none"> • <i>Mathematics: Applications and Connections: Courses 1, 2, & 3</i> 	Glencoe/McGraw-Hill
	Alternative	<ul style="list-style-type: none"> • <i>Connected Mathematics</i> 	Dale Seymour

4.2. ANALYSIS METHODS FOR IDENTIFICATION OF TASKS

Drawing heavily on the work of the QUASAR Project (e.g., Smith & Stein, 1998; Stein et al., 1996; Stein & Smith, 1998; Stein et al., 2000), we use the term *probability task* (or simply *task*) to refer to an activity, exercise, or set of exercises in a textbook that has been written with the intent of focusing a student's attention on a particular idea from probability. Any task that contained probability was considered a probability task, even if the main focus of the task was on another content area, such as geometry, combinatorics, or statistics. A probability task is not necessarily a single exercise in the textbook. A set of exercises that build on one another are considered as a single task. We have constructed such a task, as illustrated in Figure 2.

How likely is it that a chocolate chip will land on the flat side after being tossed in the air? Perform the following experiment and answer these questions to help formulate your answer to this question.

1. What are the possible outcomes for the landing position of a chocolate chip?
2. With your partner, toss 50 chocolate chips and record the landing position. How many chips landed on the flat side?
3. Based on your data, what is the experimental probability of a chocolate chip landing on the flat side?
4. As a class, pool your data. Based on the pooled data, what is the experimental probability of a chocolate chip landing on the flat side?
5. How does the experimental probability based on your data compare to the experimental probability based on the pooled data? How do you account for any differences?
6. Which of these experimental probabilities do you believe to be closest to the theoretical probability? Why? How could you obtain a better estimate of the theoretical probability?

Figure 2. Sample probability task

Likewise, a set of exercises that attend to the same topic but may be answered in isolation is considered as one task, as is the case in the task we constructed for Figure 3. Sections of probability lessons that contain narrative, such as definitions or written explanations of concepts and procedures, are not considered as probability tasks, although they are considered as portions of the textbook devoted to topics in probability.

A gumball machine contains four red gumballs, five blue gumballs, and six green gumballs. Rosalie selects one gumball from the machine at random.

1. What is the probability that the gumball is red?
2. What is the probability that the gumball is yellow?
3. What is the probability that the gumball is not blue?
4. What is the probability that the gumball is red or blue?

Figure 3. Sample probability task.

We examined each page of the selected textbooks for probability content. The portions of these textbooks that contained probability content were divided into discrete probability tasks by the first author and subsequently validated by the second author. As mentioned previously, these tasks may have consisted of several questions related to the same mathematical idea. Because of this distinction, in a given textbook the number of probability tasks that were identified was less than the number of questions, examples,

and activities related to probability. Most probability tasks were located within lessons, in both the development (e.g., worked examples, activities) and assignment portion of lessons. Other probability tasks were not located in lessons, but in chapter reviews, assessments, and extension or enrichment activities.

4.3. CODING AND ANALYZING THE LEVEL OF COGNITIVE DEMAND OF PROBABILITY TASKS

We coded each task according to the level of cognitive demand that it required. According to the Levels of Demand criteria (Smith & Stein, 1998; see Figure 1), we indicated whether the task required Memorization (Low-M), Procedures without Connections (Low-P), Procedures with Connections (High-P), or “Doing Mathematics” (High-D). Tasks containing multiple questions were analyzed as a whole; therefore, we coded each task as requiring a single level of cognitive demand. The two researchers performed check-coding (Miles & Huberman, 1994) on tasks from two randomly selected textbooks from our sample by independently coding each task and then comparing assigned codes. Initial agreement was reached on the assignment of approximately 82% of the tasks, and 100% agreement was reached after discussion. The first author then proceeded in coding all probability tasks contained in the remaining textbooks in the sample.

5. RESULTS

5.1. NUMBER OF PROBABILITY TASKS IN EACH SERIES

Figure 4 displays the number of probability tasks for each series by grade level. Note that most series have the greatest number of probability tasks in the 8th grade textbook and the least in the 6th grade textbook, although this is not uniformly the case. The Standards-Alternative series has quite a different composition, with over half of the probability tasks located in the 7th grade textbook.

There were approximately equivalent numbers of probability tasks in the New Math-Popular, New Math-Alternative, Back to Basics-Popular, Back to Basics-Alternative, and Problem Solving-Popular textbook series. The Problem Solving-Alternative series had the fewest number of probability tasks (42) of all textbook series in the sample, less than half of the number in the New Math-Alternative and Back to Basics-Popular series, which ranked next to lowest in number of probability tasks. The number of probability tasks in five of the six textbooks from the Standards era was nearly equivalent to or greater than the number of tasks in an entire series from any of the other three eras. Furthermore, it should be noted that more than half of the probability tasks from the entire sample were located within textbooks from the Standards era.

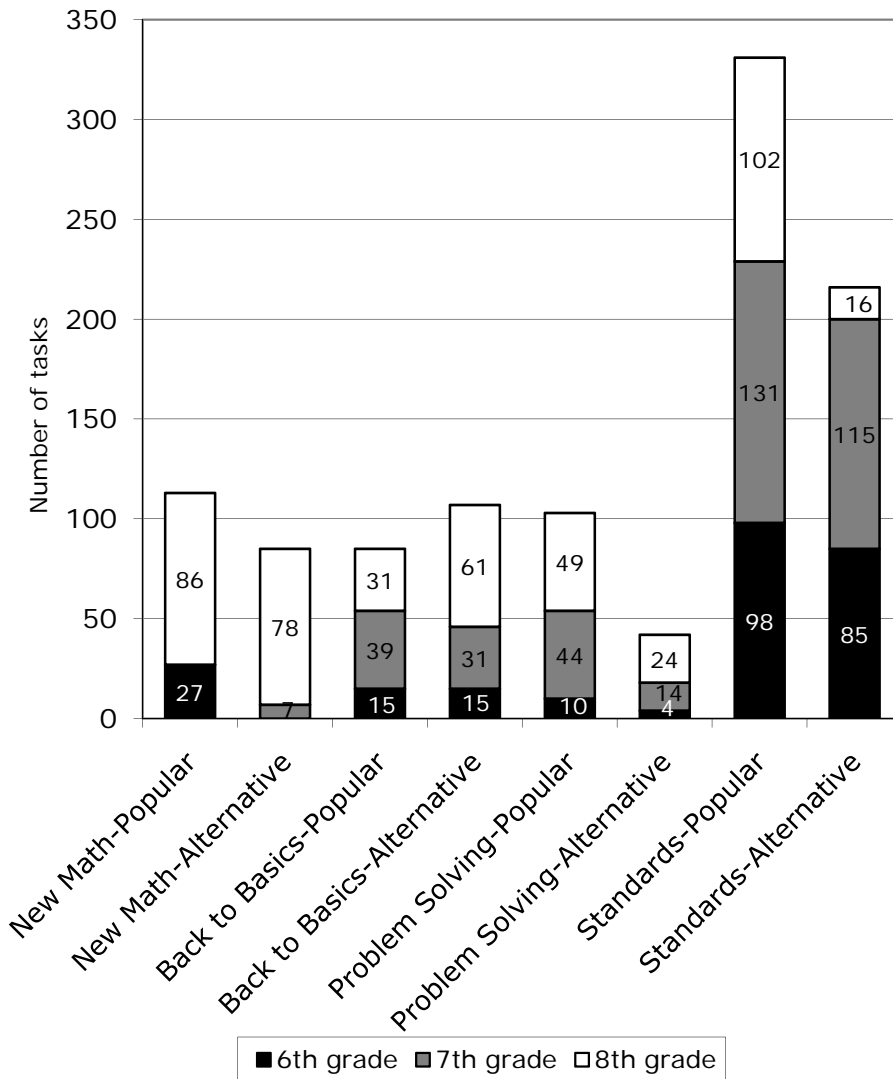


Figure 4. Number of probability tasks in each series, by grade level

5.2. DISTRIBUTION OF REQUIRED LEVELS OF COGNITIVE DEMAND WITHIN A TEXTBOOK SERIES

Within each era, and across the eras, the majority of tasks required low levels of cognitive demand, predominantly Procedures without Connections (Low-P). The Standards-Alternative series was an exception, with the majority of tasks requiring high levels of cognitive demand. The two tasks shown in Figure 5 both require lower levels of cognitive demand, Memorization (Low-M) and Procedures without Connections (Low-P). The rationale for this coding follows in the next few paragraphs.

The task in Question 3 was coded as Memorization because it was preceded by text that contained the definition of the term “dependent events,” a description of the procedure and a worked example incorporating the procedure for finding the probability of the occurrence of two dependent events. When working through the text sequentially, a student would first read the definition and worked example, and later read this task,

prompting him or her to merely recall and provide the definition. This task could be completed by referring to the preceding text, or from memorizing the given procedure of “multiply the probability of the first event by the probability of the second event” (Collins et al., 1998c, p. 522).

Questions 10 through 14 were identified as a single task and coded as Procedures without Connections. This task is algorithmic, and there is little ambiguity on how to complete the task. As described above, the text that precedes this task includes a description and worked example of the exact procedure that is to be followed. Furthermore, this task requires no explanations and appears to focus more on correct answers than fostering the development of mathematical understanding.

<p>3. Tell how to find the probability of two dependent events.</p> <p>-----</p> <p>In a bag there are 5 red marbles, 2 yellow marbles, and 1 blue marble. Once a marble is selected, it is not replaced. Find the probability of each outcome.</p> <p>10. a red marble and then a yellow marble</p> <p>11. a blue marble and then a yellow marble</p> <p>12. a red marble and then a blue marble</p> <p>13. any color marble except yellow and then a yellow marble</p> <p>14. a red marble three times in a row</p> <p>From <i>Mathematics: Applications and Connections, Course 3</i> © 1998, Collins, Dristas, Frey-Mason, Howard, McClain, Molina, et al. Published by Glencoe/McGraw-Hill. Used by permission.</p>
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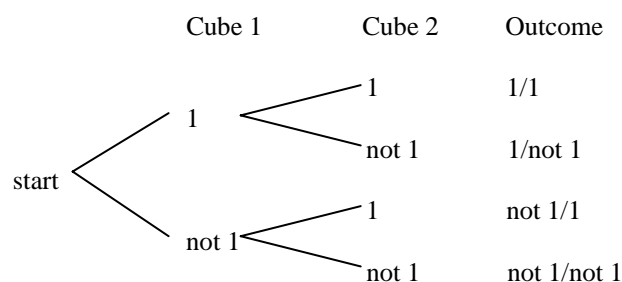
Figure 5. Examples of tasks that require low levels of cognitive demand

As stated previously, the majority of tasks in seven of the eight textbook series required low levels of cognitive demand, primarily Procedures without Connections. In contrast to these series, most probability tasks in the Standards-Alternative series required higher levels of cognitive demand. Examples of two tasks from this series requiring higher levels of cognitive demand are shown in Figure 6. The rationale behind this coding follows below.

The task in Question 19 was coded as Procedures with Connections (High-P) because it addresses a common misconception (all outcomes are equally likely) without suggesting a pathway to the solution, either in the task itself or on preceding pages. This task utilizes multiple representations of the problem situation (a counting tree and an organized listing of the complete sample space), and requires some degree of cognitive effort, as no algorithm or procedure has been previously given that addresses this situation in this form.

The task in Problem 6.1 and Problem 6.1 Follow-Up, coded as “Doing Mathematics” (High-D), requires complex and nonalgorithmic thinking. Prior to this task, the textbook authors have provided tasks to allow students to conduct experiments and calculate and compare experimental and theoretical probabilities, but this is the first request to *create* a simulation. Students are required to utilize their prior knowledge and apply it to this task. Furthermore, this task requires that students understand several mathematical concepts, including the set of possible outcomes of this game, the expected number of wins in 100 trials, and the fairness of the game. Finally, students are instructed to provide justifications for their reasoning in questions 2 and 3.

19. Tricia wants to determine the probability of getting two 1s when two number cubes are rolled. She made a counting tree and used it to list the possible outcomes.



She says that, since there are four possible outcomes, the probability of getting 1 on both number cubes is $\frac{1}{4}$. Is Tricia right? Why or why not?

Tawanda's Toys is having a contest! Any customer who spends at least \$10 receives a scratch-off game card. Each card has five gold spots that reveal the names of video games when they are scratched. Exactly two spots match on each card. A customer may scratch off only two spots on a card; if the spots match, the customer wins the video game under those spots.

Problem 6.1

If you play this game once, what is your probability of winning? To answer this question, do the following two things:

- Create a way to simulate Tawanda's contest, and find the experimental probability of winning.
- Analyze the different ways you can scratch off two spots, and find the theoretical probability of winning a prize with one game card.

Problem 6.1 Follow-Up

- If you play Tawanda's scratch-off game 100 times, how many video games would you expect to win?
 - How much money would you have to spend to play the game 100 times?
- Tawanda wants to be sure she will not lose money on her contest. The video games she gives as prizes cost her about \$15 each. Will Tawanda lose money on this contest? Why or why not?
- Suppose you play Tawanda's game 20 times and never win. Would you conclude that the game is unfair? For example, would you think that there were not two matching spots on every card? Why or why not?

From *Connected Mathematics: What Do You Expect? Probability and Expected Value* © 1998 by Michigan State University, Lappan, Fey, Fitzgerald, Friel, and Phillips. Published by Pearson Education, Inc., publishing as Pearson Prentice Hall. Used by permission.

Figure 6. Examples of tasks that require high levels of cognitive demand

Table 3 displays the percentage of tasks coded at each level of cognitive demand for each series. Using the Mann-Whitney U test (Hinkle, Wiersma, & Jurs, 1988; also named the "Mann, Whitney, and Wilcoxon test" in Hogg & Tanis, 1993), we determined that the distributions of required levels of cognitive demand were not significantly different for the three textbooks within a given series, with $p > 0.18$ in each case. Accordingly, the data presented in Table 3 represent all tasks in the series, without disaggregating by grade level.

Table 3. Percentage of tasks coded at each level of cognitive demand

	New Math		Back to Basics		Problem Solving		Standards	
	Pop.	Alt.	Pop.	Alt.	Pop.	Alt.	Pop.	Alt.
High-D	0	0	0	4	0	0	2	12
High-P	6	14	0	22	2	0	15	47
Low-P	83	82	95	74	97	98	75	40
Low-M	11	4	5	0	1	2	8	1

Note that the Back to Basics-Alternative series did not contain any tasks coded at the Memorization level (Low-M), and three series contained tasks coded at the highest level—“Doing Mathematics” (High-D). Furthermore, the Back to Basics-Popular and Problem Solving-Alternative series contained no probability tasks that required high levels of cognitive demand.

The composition of tasks found at each level of cognitive demand was very similar for the two series in the Problem Solving era; the New Math era series were also similar in this composition. In terms of cognitive demand, the series within both the Back to Basics and Standards eras appeared to be quite different, with the alternative series tending to have greater proportions of tasks that require higher levels of cognitive demand than the contemporary popular series.

5.3. TRENDS IN REQUIRED LEVELS OF COGNITIVE DEMAND OVER TIME

Typically, the most common level of cognitive demand required by probability tasks was Procedures without Connections (Low-P). The Standards-Alternative series was an exception, with nearly half of all tasks coded as Procedures with Connections (High-P). The majority of tasks that required high levels of cognitive demand were located within the series of the Standards era, as were the majority of tasks that were analyzed for this study. More specifically, there were a greater number of tasks, but not necessarily a greater percentage of tasks, that required higher levels of cognitive demand in the two Standards-era textbook series. The Standards-Popular series is a case of this phenomenon. It contains more tasks requiring higher levels of cognitive demand than any series from a previous era, but a smaller percentage of higher level tasks than the Back to Basics-Alternative series.

6. DISCUSSION

6.1. INTERPRETATION OF INCREASED NUMBER OF TASKS IN MORE RECENT SERIES

There was a dramatic increase in number of probability tasks in the textbooks from the Standards era, compared to the three previous eras of mathematics education. In particular, over half of all of the tasks analyzed in this study were located within textbooks from the Standards era. This increase in attention to probability appears to have coincided with the release of national recommendations such as NCTM (1989) that advocated the inclusion of probability in the middle grades. Although the design of this study did not allow for the identification of causal factors, the proliferation of probability tasks within the Standards era appears to be consistent with the contemporary recommendations for the inclusion of probability topics within the middle grades mathematics curriculum.

6.2. INTERPRETATION OF STABILITY OF DISTRIBUTION OF LEVEL OF COGNITIVE DEMAND WITHIN EACH SERIES, BUT DIFFERENCES BETWEEN SERIES

As stated above, across the three grade levels of each series, the distributions of required levels of cognitive demand of probability tasks were not significantly different. For most series, probability tasks required predominantly low levels. In the Standards-Alternative series, however, a majority of probability tasks (59%) required high levels of cognitive demand. Therefore, the Standards-Alternative series adhered to the recommendations of Stein et al. (2000) that students at each grade level should have opportunities to “engage with tasks that lead to deeper, more generative understandings regarding the nature of mathematical processes, concepts, and relationships” (p. 15). Furthermore, the use of tasks that require higher levels of cognitive demand in instruction supports the development of conceptual understanding that is called for by the NCTM (1989, 2000).

It is likely not a coincidence that the series with the highest distribution of required levels of cognitive demand (Standards-Alternative) was the same series that received the highest quality ratings in the Project 2061 study of mathematics textbooks for middle grades students (AAAS, 2000). The Project 2061 study did not examine the treatment of probability, but instead focused on number, algebra, and geometry. Although numerous criteria were used to render quality ratings, the results from our study indicate that the probability portions of this series may be of similar high quality. Additionally, Project 2061 researchers found that two other series included in our study (Standards-Popular and a revised edition of Problem Solving-Alternative) were of lower overall quality than the Standards-Alternative series; their findings coincide with results from our study that the probability tasks contained in the Problem Solving-Alternative and Standards-Popular series required significantly lower levels of cognitive demand than the Standards-Alternative series. Although the distribution of required levels of cognitive demand is not equivalent to the quality of instruction in a textbook, these measures are similar in that they address the potential opportunities for students to develop deeper understandings of mathematical content.

In the Back to Basics and Standards eras, there were significant differences in the distribution of required levels of cognitive demand for probability tasks between the popular and alternative series ($U = 3199.5$, $Z = -5.443$, $p < 0.001$ and $U = 19661.5$, $Z = -10.273$, $p < 0.001$, respectively). In each case, the alternative series had a higher distribution of required levels of cognitive demand. This may reflect the desires of the authors to offer something truly different. These alternative series presented more than a new sequence of topics or additional topics; indeed, the *nature* of the tasks within these textbook series was substantially different. This lends credence to the notion that these series represented true alternatives to the contemporary popular series.

7. RECOMMENDATIONS AND IMPLICATIONS

With the exception of the Standards-Alternative series, the vast majority of tasks in each series were characterized as requiring low levels of cognitive demand, usually at the Procedures without Connections level. Indeed, *all* tasks in the Back to Basics-Popular and Problem Solving-Alternative series were coded as Procedures without Connections, save five tasks within the Back to Basics-Popular series and one task in the Problem Solving-Alternative series coded as Memorization. Stein, Grover, and Henningsen (1996) found that the level of cognitive demand of a task as written tends to either stay the same or

decline when implemented by the teacher. For this reason, textbooks should include tasks that require high levels of cognitive demand, and thus provide potential opportunities for students to experience mathematics as more than a set of unrelated procedures and facts. The inclusion of tasks that require high levels of cognitive demand has the potential to foster a more connected view of mathematics as related, meaningful concepts and procedures useful for solving many types of problems.

Heretofore there is no research documenting the impact of specific curricular tasks on student learning in probability, delineating between the kinds of reasoning tasks at each level may promote. Nevertheless, it seems reasonable to conjecture that the nature of a set of probability tasks might influence students' views of probability, and promote a more classical, frequentist, or subjective approach (for a more detailed description, see Batanero, Henry, & Parzysz, 2005; Borovcnik, Bentz, & Kapadia, 1991). For example, low-level tasks, including Procedures without Connections, might promote a classical, deterministic outlook, in which students rely on calculations of theoretical probabilities with little or no appreciation for how much variability might be expected in repeated trials of a probability experiment. On the other hand, higher-level tasks, such as Doing Mathematics, might foster a frequentist view in which students grapple with disparities between empirically-derived and theoretically-derived probabilities. This latter approach not only aligns with recent curriculum frameworks (e.g., NCTM, 2000), it is also consistent with research on learning probability that advocates teachers (a) "make connections between probability and statistics," (b) "introduce probability through data," and (c) "adapt a problem-solving approach to probability" (Shaughnessy, 2003, p. 224). Indeed, there is a growing body of evidence (e.g., delMas & Bart, 1989; Pratt, 2000; Pratt, 2005; Stohl & Tarr, 2002; Yáñez, 2002) to support the role of simulations as a means of fostering more sophisticated understanding of probability concepts in place of the well-documented *equiprobability bias* (Lecoutre, 1992), *outcome approach* (Konold, 1991), and *representativeness* (e.g., Konold et al., 1993). Therefore, we argue that more recent curricular materials containing higher-level probability tasks have the potential to promote sound probabilistic reasoning, challenging some of the primary intuitions students bring to the classroom, and preparing them to make sense of the chance variation and random phenomena they will inevitably face in the real world.

Results of this study revealed differences in the extent and nature of the treatment of probability across series. Prospective and practicing mathematics teachers may benefit from applying portions of the framework used in this study to analyze curricula and, in doing so, dispel the notion that all textbooks are created alike. In particular, determining the levels of cognitive demand required by tasks revealed that for most series, the majority of probability tasks required low levels of cognitive demand. By conducting similar analyses, it is possible prospective mathematics teachers will be more prepared to scrutinize their own textbook and realize the need to increase the levels of cognitive demand of tasks. Such teacher education activities form an important part of the call from researchers for prospective teachers to critically analyze textbooks and other curriculum materials (Lloyd & Behm, 2005; Remillard, 2004). In addition, preservice teachers with experience in analyzing curricula may be better prepared to assist in the selection of curriculum in their future positions, and less likely to examine only surface characteristics of textbooks.

Only two textbook series were selected from each era for the sample of this study. The mathematics curriculum of a particular era may be more fully described if more series, particularly more popular series, from each era were examined. Furthermore, the "alternative" curriculum may be more appropriately characterized by analyzing commonly used supplemental materials that did not fit the selection criteria to be

included within this sample. For example, several mathematics educators familiar with middle grades mathematics curricula identified the *Middle Grades Mathematics Project* (Phillips et al., 1986) and *Transition Mathematics* (Usiskin et al., 1995) as widely used alternative curricula, but neither set of materials was written as a comprehensive curriculum for students in grades 6, 7, and 8. Thus, research should analyze a broader range of curricular materials, both popular and alternative, in order to more fully characterize the mathematics curriculum of particular eras of mathematics education.

For most of the series, no significant difference was found between the distribution of required levels of cognitive demand of tasks in the development and assignment portions of lessons. In each of the textbook series in the first three eras and the Standards-Popular series, $p > 0.05$. In the Standards-Alternative series ($U = 2821$, $Z = -3.03$, $p < 0.01$), the tasks in the development portions of lessons tended to require higher levels of cognitive demand than tasks within the assignment portions. Further research should examine the distribution of required levels of cognitive demand for assessment tasks as well, and compare these distributions to those of tasks within the development and assignment portions of lessons. Results from this study revealed that relatively few probability tasks were written for the purpose of summative assessment. For this reason, future research may need to examine ancillary materials for assessment items. Such analyses may reveal a potential mismatch between assessment and instruction. In particular, it may be that tasks within lessons require high levels of cognitive demand, whereas assessment tasks require low levels of cognitive demand.

Finally, this study focused on describing the intended probability curriculum as present in middle grades mathematics textbooks. Informed by the results of this study, future research should investigate the enacted probability curriculum as presented by teachers in contemporary classrooms. This enacted curriculum should include more than the particular chapters and lessons that were covered, but also which tasks were assigned, and why teachers chose to include or omit particular portions of the intended probability curriculum from their instruction. Such an analysis of the enacted curriculum was beyond the scope of this study, but it needs to be examined in order to more precisely determine students' opportunity to learn.

8. CONCLUSION

This study addressed an existing void in the research base by analyzing the treatment of probability within middle grades mathematics textbooks from a historical perspective. Moreover, it represented the first documented attempt to analyze the levels of cognitive demand required by probability tasks within textbooks published across four recent eras of mathematics education. Information about the levels of cognitive demand required by tasks within a textbook may prove to be one measure of the quality of the mathematics presented within a textbook or textbook series. Indeed, a significant result of our study was the increase in the number and proportion of tasks that required high levels of cognitive demand within the alternative textbook series from the Back to Basics and Standards eras. In essence, these two series may be viewed as models for the development of future curricula because they challenge students to move beyond the development of mere procedural knowledge of probability.

In this era of *Standards*, middle grades mathematics textbooks are devoting more attention to probability and requiring high levels of cognitive demand. Whether attributable to the recommendations of researchers or professional organizations, the alternative textbook series of the Back to Basics and Standards eras in this study demonstrate a markedly different approach to the level of cognitive demand required by

probability tasks, and the alternative series from the Standards era provided many more opportunities for students to engage in these types of tasks. What remains to be documented is the impact of such curricular materials on student learning, particularly in reference to supporting students' understanding of probability concepts, so that probability misconceptions will become less prevalent among people of all ages.

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