

THE ROLE OF LANGUAGE IN BUILDING PROBABILISTIC THINKING

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ABSTRACT

This paper is based on research that investigated the development of probabilistic language and thinking by students 10–12 years old. The focus was on the adequate use of probabilistic terms in social practice. A series of tasks was developed for the investigation and completed by the students working in groups. The discussions were video recorded and complemented by the students' written notes. The analysis was carried out under a historical-cultural perspective. We have identified how some notions about frequency, chances, possibility and probability are intuitive and how others are mistaken. Subjectivist probabilistic thinking is present in students' ideas, and this indicates the need to develop teaching approaches that confront and overcome these ideas.

Keywords: *Statistics education research; Probabilistic language; Subjectivist probability; Negotiation of meaning*

1. INTRODUCTION

The study reported in this paper belongs to a broader research project in statistics education, funded by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, National Council of Scientific and Technological Development), entitled “Knowledge in stochastics produced and mobilized by a collaborative study group”, and carried out from 2008 to 2011. The research was conducted within Grupo Colaborativo em Matemática (Grucomat, Mathematics Collaborative Group), linked to São Francisco University. The group has existed for 10 years and is made up of schoolteachers, graduate students and professors (who authored this article).

With the inclusion of statistics, combinatorics and probability in the Brazilian elementary school curriculum (students 6–14 years old), Grucomat dedicated itself to organizing tasks to be carried out in classrooms. Such tasks, created collaboratively within the group, were employed by participants in their own classrooms, and those classes were videotaped. This material together with the teachers' (spoken or written) narratives constituted the group's objects of analysis.

Probability content was regarded as the biggest challenge for teachers, since it had previously belonged only in the high school curriculum (students 15–17 years old). The group considered the question of what would be the most suitable approach for younger students. A set of tasks was proposed to address the problem, aiming to analyze conceptual development in probability, beginning with probabilistic language. Our focus on language issues is derived from our theoretical grounding in the historical-cultural perspective of Vygotsky and collaborators' studies, which consider the interrelations between thought and language.

Data presented here were collected from a group of 12 students from sixth grade (10–12 years old) from a public school in São Paulo. Students attended two sessions of analysis and

discussions about a set of tasks related to probability. From this set, two tasks were chosen to be analyzed in the present paper. Both tasks prompted higher debate among students, from the probabilistic language standpoint, as will be explained.

This article is divided into four sections: firstly, the theoretical bases are presented; secondly, the adopted methodology is described; thirdly, the written results (the students' answers to exercises) and extracts from their statements are shown; and finally, we present our conclusion about the results achieved.

2. THEORETICAL BASES

2.1. LANGUAGE FROM A HISTORICAL-CULTURAL PERSPECTIVE

Adopting a historical-cultural approach, in Vygotsky's perspective, as the theoretical basis for the research means focusing on understanding senses and meanings built and shared by the subjects, during interactive and dialogical contexts, assuming that knowledge is processed through people's interaction. Learning is seen as a social process and language plays a central role in human development of understanding. Considering the variety of conceptions of 'language', some researchers for this theoretical approach have chosen 'word' as the phenomenon of unity between speech and thought. Our interest in taking 'word' as the object of analysis for this paper is based on the fact that Vygotsky has considered it as endowed with sense and meaning. For him, the word's meaning is inconstant; it evolves both in one's development and within the historical development of the word itself.

For Vygotsky (2001, in Prestes, 2012, p. 79), "Word meaning is a phenomenon of thought only in so far as thought is embodied in speech, and of speech only in so far as speech is connected with thought and illumined by it." It is a dynamic process; thought embodied in speech.

We agree with Góes and Cruz (2006, p. 34) that "although word meaning is always a generalization act, it modifies itself as the child faces new situations in which to use the word, and as his/her intellectual processes of abstraction and generalization improve." So, until a child or student understands the meaning shared between the members of a certain social environment, the meaning of a word alters considerably for them.

While the word meaning is being modified, it accumulates one's intellectual and affective constructions, acquiring a personal aspect – this would be the understanding of the meaning. It can be said that meaning is just a part of sense. In the socially-accepted meaning of a word, the context within which it is used is taken into account. The word contains the speaker's internally-assumed experience, and acquires its meaning in the concrete context of speech. Meaning changes

alongside a child's development. It is also changed by different ways of thought functioning ... word is not solely related to a certain object, but to an entire class of objects. That is why each word represents an occult generalization; each word is already a generalization. (Prestes, 2012, p. 80)

Considering the inseparability between sense and meaning, we have opted to use the term 'signification', understanding that it better represents the fluctuation between the stable meaning of the word and its extremely variable sense. So, we also understand that the classroom, whether in group discussion or when sharing ideas and collaborative work among students, is an environment for learning and producing signification in created discourses.

Understanding the dynamics of producing and negotiating signification implies paying attention to students' conceptual elaboration process, understood as a dynamic and evolutionary process.

At any age, a concept embodied in a word represents an act of generalization. But word meanings evolve. When a new word has been learned by the child, its development is barely starting. ... The development of concepts, or word meanings, presupposes the development of many intellectual functions: deliberate attention, logical memory, abstraction, the ability to compare and to differentiate. These complex psychological processes cannot be mastered through the initial learning alone. (Vygotski, 2001, p. 248)

Understanding students' conceptual elaboration process implies considering the intrinsic relation between everyday (or spontaneous) concepts and scientific ones. For Vygotski (2001), both concepts are developed in opposite directions, but are interdependent; both are products of school instruction, developing as a network of relationships with other concepts.

Adopting a historical-cultural approach implies considering that the role of school education is to develop the scientific thought of children and teenagers. In this process, we must consider movements between everyday concepts and scientific ones, as well as the adult teacher's role in classroom negotiation, in the dialogic relation among the participants.

The earlier maturation of scientific concepts is explained by the unique form of cooperation between the child and the adult that is the central element of the educational process; it is explained by the fact that in this process knowledge is transferred to the child in a definite system. This is also why the level of development of scientific concepts forms a zone of proximal possibilities for the development of everyday concepts. The scientific concept blazes the trail for the everyday concept. It is a form of preparatory instruction which leads to its development. (Vygotski, 2001 in Prestes, 2012, p. 189)

Our goal is to examine this movement of conceptual elaboration carried out by students as they learn probability, taking 'word' as the unit of thinking. To this end, we also considered studies in statistics education that investigate the development of probabilistic language and thinking.

2.2. PROBABILITY LANGUAGE

Many difficulties and misunderstandings shown by students when trying to solve problems in probability are initially related to the lack of a suitable language with which to interpret the problems. We are guided here by the conclusion presented in the research of Bentz, Borovcnik, and Bentz (in Sáenz, 1999), which argues that answers given to the problems may not represent the students' thought, because language-related questions might confuse them or even reinforce their mistaken beliefs about probability. Green's research (also in Sáenz, 1999) highlights students' inadequate verbal ability to describe probabilistic situations coherently. Other studies (Amir & Williams, 1999, and Green, 1983, (both) in Jones, Langrall & Mooney, 2007) have investigated students' conceptions of the word 'chance'. Most of them talk about several meanings, such as: "something that just happens", "something unexpected", or "an unusual event". According to Jones, Langrall, and Mooney (2007, p. 917):

When students quantify chance situations, evidence suggests that they misuse language.

The most common example is use of the phrase 50-50 to describe the chance of a particular event. In situations in which an event is possible, students frequently use the expression 50-50 chance to indicate the presence of uncertainty rather than a specific measurement of chance.

For these authors, children's and teenagers' idea of uncertainty and chance must be dealt with in school from early ages, so they can develop a suitable language to address their misconceptions about chance and probability. Many students believe that events are always equally likely, a notion that leads to a fifty-fifty representation of each situation.

Probability classes in high school barely approach the negotiation of meaning of terms such as 'certainty', 'uncertainty', 'possible', 'less probable', 'improbable', and so on. It is presumed that these terms are already part of students' vocabulary. Yet, from students' different interpretations and uses of such terms, it becomes evident that it is necessary to deal with probabilistic language while developing concepts of probability. Conceptual issues require a language for their expression, but this very language enables the creation of new concepts or the reformulation of misconstrued concepts.

We have identified several discussions in the literature concerning conceptions of probability. Godino, Batanero, and Cañizares (1996) and Fernandes (1999), for example, divide them into four categories:

- (1) the classical concept,
- (2) the frequency or empirical concept,
- (3) the subjectivist concept,

(4) the axiomatic or formal concept.

The main feature of the frequency or empirical concept is that an event's probability emerges from an experimental process. According to Godino, Batanero, and Cañizares (1996), the value of a probability is given by the relative success obtained in experiments. Probabilities are based on the results of real experiences, in what is called "probability a posteriori", when probability is calculated after experiments. Interpretation of such probability can lead to mistaken decisions; such mistakes result from using relative frequency as a limiting value, and also from different interpretations of "randomness" and "similarity" (Fernandes, 1999).

From the subjectivist point of view, probabilities express the degree of personal belief or perception. A person uses his or her experience and knowledge of the situation to express how probable an event could be, which can lead to different values for the probability of the one event.

In this paper, our focus is on the frequency and subjective approaches, as the basis of a pedagogical effort to develop a theoretical conception of probability, since other studies carried out at Grucomat (Furlan, 2011; Marocci, 2012; Santos, 2010) have revealed that in the early phases of elementary school studies of probability, these approaches are predominant.

In this sense, our studies come close to Watson's ideas (2006). According to her, three conceptions of probability – frequency, subjective, and theoretical – must be introduced in the early years of school education, but activities related to the frequency and subjective approaches will provide a sound foundation for the theoretical approach. According to Watson, it is a pedagogical mistake to avoid activities based on the first two conceptions when it comes to the third, because of students' conflicts about notions of chance in the school context and beyond. To involve students in statistics literacy, contact with probability must originate in real-world situations, returning to it in such ways as to resolve those conflicts. Further, Watson (2006) suggests that notions of probability should be introduced to younger students by using chance-related language – terms such as 'possible', 'impossible', 'certain', 'uncertain'.

Therefore, we conclude that Watson's ideas (2006) might be supplemented with the historical-cultural perspective of 'word' as a tool to explore the relationships between thought and probabilistic language. We believe that different conceptions are present in the ideas and speech of students in elementary education, particularly those who have not had the opportunity to explore probability theoretically as a measure of chance.

3. METHODOLOGY

The historical-cultural approach is directly related to the qualitative approach chosen to conduct this research study. We understood research as a social practice constructed by the researcher, from the production of empirical data to analysis; the outcome is one possible interpretation and always subject to change. We agree with Smagorinsky (2011, p. 60):

Empirical research itself is a social construction, being developed and practiced primarily in Western cultures that value the development of 'scientific' thinking (Vygotsky, 1997); that is, the development of formal concepts that are abstracted from the immediate context of their usage and used to develop formal rules for broader application than is available from the setting of initial learning. Empirical research thus typically strives for generalizability from the evidence found in the context of particular studies, with principles derived for application to other similar situations, if not to all situations.

Furthermore, we endorse the view that the environment of investigation must be ruled by the interactive and dialogic movement between agents – between teachers and students and among students themselves. In this sense, the tasks we have created or adapted are always based on students working in groups, socializing ideas that emerge from their discussion.

Before utilizing the tasks in the classroom, we decided to hold two sessions with elementary school students in which our goal was to identify the meaning that they would attribute to terms in probabilistic vocabulary. A partnership was established with a school and a group of sixth-grade students (10-12 years old) who were invited to collaborate on the research study. Twelve students volunteered and, after securing their parents' approval, we conducted two sessions with them at the university.

These students formed themselves into three groups, and each group included a teacher, who observed the discussions, made interferences, introduced problems, and recorded the interactions. The small-group working sessions were recorded, and the whole-group discussions, mediated by the researcher, were videotaped. A researcher was responsible for putting together the results of the tasks carried out by the three groups, helping them to negotiate the meaning of various words. We have focused here on four words: frequency, possibility, probability and chance. The groups were:

Group 1: Fernanda, Taiane, Larissa, and Thainara.

Group 2: Sabrina, Isadora, Rúbia, and Francine.

Group 3: Henrique, Kaio, Vítor, and Christian.

The data we present are related to the written records made by the students during the group discussions (Task 2), and to the video recordings made during the whole-group discussion period (Tasks 1 and 2). In this way, the group's conclusions were negotiated collectively, with the mediation of the researcher. Obviously, during the collective discussion time, some students' voices were louder than others', especially those chosen by their colleagues to be the group's presenter. The videotaped images enabled us to investigate students' gestures as well as speech. Gesture observation is necessary to understand the development of the whole class, as well as that of some students who did not express themselves verbally, which does not imply lack of participation.

Using two tasks (given in Appendix A), our objective was to offer students contact with and reflection on words that belong to our everyday life as well as to probabilistic language, since we use them to express our beliefs regarding various events. Our purpose was to observe how students added meaning to the words, exploring the relationship between our ideas and those of Watson (2006).

4. RESULTS AND DISCUSSION

On task 1, there was almost no disagreement among groups as to the words' meanings. Most of them linked the columns in this (not completely correct) way:

- *Cannot occur* ↔ *Improbable*
- *Does not occur very often* ↔ *Less Probable*
- *Occurs frequently* ↔ *Very Probable*
- *Occurs almost always* ↔ *Probable*

Divergence took place in Group 2:

- *Cannot occur* ↔ *Less Probable*
- *Does not occur very often* ↔ *Probable*
- *Occurs frequently* ↔ *Improbable*
- *Occurs almost always* ↔ *Very Probable*

In group 2, students demonstrated that they perhaps did not understand the meaning of 'probable' and 'improbable'. They described an event that 'cannot occur' as 'less probable', a phrase which implies that it can still occur. The same happened as the phrase 'occurs frequently' was linked to the term 'improbable'. In this case, we believe that they had not understood the meaning of 'frequently'. If it does occur often ('frequently'), how could it be improbable?

Disagreement regarding the matching of terms was raised during the socialization time: groups tried, through their arguments, to convince the others that their statements were correct. Yet, two students from group 2 did not change their previous statement. When the researcher realized that the issue was the meaning of the word 'frequently' she had the following exchange with the students (see Appendix B for the original Portuguese):

Excerpt 1

Researcher:	What is a frequent event?
Henrique:	It occurs frequently, all the time, for example, people get hit by cars.
Researcher:	Do people get hit by cars all the time?
Henrique:	Not every day.
Researcher:	So it is frequent, but not every day?
Henrique:	It can be frequent, but I don't know [if it is going to happen].
Researcher:	What is a frequent event which doesn't happen almost always? ... Let me give an example: World Cup is a frequent event?
Students:	No!
Researcher:	Why not?
Vitor:	Because it doesn't happen every year.
Researcher:	What is the World Cup frequency?
Students:	Every four years!
Researcher:	And does it occur almost always? How many World Cups have you watched?
Vitor:	Two. One I was 4 years old and the other I was 8 years old.
Henrique:	Frequency is something that can't happen always.

Initially, the researcher examined the meaning the students gave to the word 'frequency'. There was an expectation that students would relate it to everyday contexts, since conceptual elaboration implies establishing connections between spontaneous and scientific concepts (Vygotsky, 2001).

In this process of negotiating the meaning of the word, the researcher uses the very speech of the students and inserts a problem into it. When realizing that the discussion implied an assumption that something must happen every day to be frequent, she used an example that breaks this conception: the frequency of the World Cup. We later realized that the example was not suitable since – for the researcher – the World Cup had a frequency, but for students it had happened only twice. This situation prompted us to consider that in the mediation process, teacher/researcher subjectivity may not contribute significantly to exemplifying situations for students, offering only limited possibilities to advance the students' signification process.

Our experience has shown that during discussion time, when the negotiation of word meaning seems stuck, it is better to try another strategy. So, the researcher chose to discuss Task 2, since it could introduce elements that would enable students to revise their ideas. We found that by doing Task 2, group 2 could rethink their solution to Task 1.

Table 1 shows a synthesis of students' conceptions, a written record of the meaning of words proposed on Task 2. As the presenter from each group read aloud the phrases created by their groups, the researcher wrote them down on the blackboard.

Table 1. Synthesizing of students' answers to Task 2

Words	Meanings
Impossible	<i>It will never happen. It will never be possible to happen. It's raining money. It's when you won't be able to do it. Became rich from one day to another. A tree that bears money. It is impossible to reach the center of the Earth. It can't be done. Money falling from sky. It can't happen.</i>
Possible	To visit Universidade São Francisco. <i>It might happen someday.</i> An astronaut arrives to Mars. It is possible someone plants several trees and someday we are aware that, besides helping our neighbors, we are helping the environment. <i>When you will make it.</i> Winning the lottery. <i>It will always happen.</i> Become rich. <i>Something that may occur</i> , for example, a motorcycle and a car crash.
Equally possible	<i>It is almost possible to occur.</i> A virus has spread around the world. <i>When possibilities are always equal.</i> In an exam, all students have the possibility to get the same grade. All the students in the classroom could be chosen to come here (to the university). In a challenge, students could get the same grade. I might get rich and my friend too. We all have the possibility to pass grade.

Less possible	Winning the lottery. <i>When possibilities are not equal.</i> Not all can go out for a drive. It is less probable that a person does not answer anything. That I get an A-grade. <i>It almost never happens.</i> A crazy man running out from a madhouse.
Very possible	<i>It is probable to happen. Possible to happen. Very probable.</i> It is very probable that AIDS might kill. <i>It has a lot of chance to happen.</i> Something that might happen someday. I might get rich tomorrow. It is very probable that I will have a better future.
What do you think probability means?	Something is possible when it has a lot of chances to happen. Something that has a lot of chance to happen. It might happen. It has a chance to happen or not. Something that might happen or not. It might exist or not.

In Table 1, the phrases in italics were used by students to indicate the meaning of words, in addition to the examples. We found that as a result of our giving examples, they used everyday situations to try to attribute meaning to these words within familiar contexts, such as: winning the lottery, raining money, getting a virus, visiting the university (an unlikely event for most Brazilian students from the working classes). To Watson (2006, p. 128), “Intuitions and subjective beliefs are the starting points for a chance curriculum and these are usually expressed through language rather than numbers.”

So, students’ subjective concept of probability was demonstrated. As we said previously, this conception expresses personal beliefs or perceptions based on experiences, knowledge and interests in a context. According to Fernandes (1999), it is a “personal” conception, based on personal evaluation of random situations. As in Watson’s studies (2006), the responses given by students to the words in Table 1 suggest ideas related to the contexts and/or explanations about the meaning of the words – as highlighted in the table. For the researcher, this is a fundamental discussion: “exploring contexts where students believe random happenings take place is an important foundation for later work” (Watson, 2006, p. 131). In her studies, Watson has found that younger students relate words (such as ‘chance’, ‘possibility’, and ‘probability’) to the school environment, while older ones relate them to a broader social context. In our study, we found that these students use both contexts.

Since the words ‘possibility’, ‘probability’, and ‘chance’ also showed up during the second task, perhaps because they were previously discussed, the researcher re-emphasized them during the collective discussion. The words ‘impossible’, ‘possible’ and ‘equal possibility’ did not generate conflict or dissonance. The most intense discussion occurred around the fourth expression, ‘less possible’.

By noting the phrase ‘winning the lottery’, given by group 1, the researcher could exploit the idea of an impossible event; Larissa’s immediate answer demonstrated her comprehension of this kind of event:

Excerpt 2

Researcher:	If I don’t play the lottery, is there less possibility in winning?
Larissa:	If you don’t play, there is no possibility.

Immediately after, there was a discussion about the words ‘possibility’ and ‘probability’, since both are used to express ‘very possible’ according to row 5 of Table 1. Students returned to their earlier discussion about the cure for AIDS. The researcher asked a question which Larissa answered, referring to the cure for AIDS:

Excerpt 3

Researcher:	So, if I say it is very probable and very possible, is there any difference?
Larissa:	Probable has treatment; possible doesn’t.

The context in which Larissa uses the words ‘probable’ and ‘possible’ returns to the previous discussion about spread of a virus, such as AIDS (referred to on the third row of Table

1). This shows how ideas can go away and return when negotiating the meaning of words, looking for new significances. In terms of students' misconceptions, the teacher/researcher did not always grasp an interesting quote that might generate new meanings, such as Larissa's. Only later, when analyzing data, did we realize that, while the researcher was looking for the difference between the concepts of 'possibility' and 'probability', Larissa was still using the context language (Watson, 2006). During the discussion, the researcher did not realize this peculiarity and insisted on investigating the difference, leading to a re-iteration of the fundamental question:

Excerpt 4

Researcher:	Can I use 'possible' instead of 'probable'?
Henrique:	No
Researcher:	Why? What is probability?

We know that during preliminary work with probability it is not expected that students of this age would have a formal conceptualization about what probability is. However, according to Vygotski (2001), a word carries within it its inconstant meaning, which can be developed, a link between thought and language. He writes: "Thought is not expressed but completed in the word" (Vygotski, 2001, p. 409). We therefore understand that using a word in pedagogical practice implies negotiating its signification, setting ideas into motion and enabling progress in the levels of generality of concepts.

The researcher's question led the group to discuss the last item of Table 1, "What do you think probability means?"

Excerpt 5

Larissa:	Something that has many chances to happen. <i>[The word 'chance' is used for the first time.]</i>
Researcher:	Something that has many possibilities to happen? So, Larissa, you think that probability is the same as possibility?
Larissa:	Yeah!
Henrique:	No! Probability is when I say 'half-half,' 'fifty-fifty'. It is mathematics. Possibility is not a number, it is not math, it is more language. It doesn't have measure; you don't have to say 50%.
Researcher:	What do you think? Does what Henrique said make sense? <i>[The researcher opens the dialogue to the group, once there is a conflict between what Larissa and Henrique said.]</i>

After some interference of students, it seemed to be consensual that to be probable it would be necessary to be quantified and equally likely, adding that to their notion of being probable, implying that it must have a lot of chance to happen. The researcher took the word 'chance' and asked, "What does chance mean?"

Excerpt 6

Larissa:	Larissa: It is when, for example, it has 50% chance of happening. 70% chance of having a better world.
Researcher:	Larissa, when you are talking about chance, you talk about numbers, measurements.
Larissa:	No, I am just giving an example.
Henrique:	The chance might be of 70% and 30%.
Researcher:	Now I am confused, Henrique! You had said that to be probability, it had to be 50% and 50%. What about chance?
Henrique:	Probability must be the same, chance, not.

Aiming to end the meaning negotiation, the researcher suggests:

Excerpt 7

Researcher:	If the six of us [the four students and two researchers] play something, what is the chance of any one of us winning?
Isadora:	One in six. And if we don't play, there is no chance.

It became clear from this discussion that for some students the words ‘possibility’, ‘probability’ and ‘chance’ have a very similar meaning when used to describe everyday issues. But when those students are asked about the difference between the terms, they reinforce the idea that ‘probability’ refers to a measurement of equally-likely events, as Jones, Langrall, and Mooney (2007) found in their research. Henrique was the student who most emphasized this idea, by saying “half-half,” “fifty-fifty”. He went further in his explanation, saying that probability involves a measurement, because it is mathematics. As for the word ‘chance’, they think that measurements don’t need to be equal, and so don’t represent equally likely events.

It is important to highlight that, although our objective was to negotiate the meaning of the words in a practical session that preceded any quantification, the researcher’s question “What do you think probability means?” mobilized students to reflect about the difference of meaning among these words. It enabled the discussion between Larissa and Henrique about different conceptions of possibility and probability when the word ‘chance’ was included in the discussion. Quantifications do not appear in the written record of the discussion, but they do appear in Henrique’s speech, when he disagreed with Larissa that probability and possibility have the same meaning. This reflects Henrique’s need to quantify in order to attribute meaning to words. His intervention mobilized the entire group, especially Larissa, to think about the meaning of the word ‘probability’ related to quantification. We emphasize that the quantification aspect brought up by Henrique is still within the school context.

The circulation of ideas within the school context, mediated by a skilled adult (researcher) will enable the students’ conceptual development. Our objective for this study was not to identify how each student developed during the discussion, but to emphasize how a process of negotiating meaning of words contributes to the collective learning – of the students, and also of the teacher/researcher.

5. IMPLICATIONS FOR RESEARCH AND PRACTICE

The results shown above have confirmed our initial hypothesis, built from the literature, that conceptualization requires a negotiation of the meaning of words from a probabilistic vocabulary, since many of them are known and socially used by students. Their meaning needs to be re-established within the context of a conceptual elaboration to acquire scientific knowledge.

Tasks with probabilistic language, when carried out before the resolution of problem-situations, offer to students a linguistic repertoire that will allow them to provide justifications for their solutions. Words like probability, possibility, chance, probable, possible, certainty, etc., are used by students with the social context as background. Therefore, a variety of contexts must be created to enable the signification processes. Some results point to how much the understanding of words such as frequency, probability, possibility and chance, among others, can facilitate the expression of a probabilistic thought while also indicating mistaken interpretations or meanings in contexts that demand decision making, as pointed out by Watson (2006):

The number of connections to fundamental concepts across the mathematics curriculum, as well as links across the school curriculum and outside of school where uncertainties abound, make chance an important part of the curriculum. The necessity to appreciate the nature of chance for decision making in many contexts outside of school makes it an important contributor to statistical literacy (Watson, 2006, p. 128).

It is during the dialectical movement of significations that students enter the process of conceptual elaboration; concepts acquire signification and levels of generality are increasingly refined. We can conclude, based on these initial investigations, that those tasks related to probabilistic language and involving interaction between students and teachers, as well as those related to broader contexts, enable the development of students' probabilistic thought.

We agree with Shaughnessy (1992) that it is necessary to create space within the classroom for the coexistence of subjective conceptions (such as those shown here) with empirical and frequency conceptions of probability, engaging students in intentional work that would allow the theoretical conceptions to emerge.

The data presented also demonstrate the importance of collaborative work between students and the processes of negotiating meaning, because it is within these dynamics that students learn, and thought is materialized into words. Although not all students have taken turns in the verbal discussions, the video analyzed by us shows their silent participation, agreeing or disagreeing with the presenter by gestures.

In this way, students develop and reach other levels of scientific knowledge. Such evidence confirms some principles of the historical-cultural approach, according to which the school's role (the intentional instruction) is to promote the human development in its collectivity.

What the child is able to do in collaboration today he will be able to do independently tomorrow. It means that, when verifying the possibilities of children utilizing collaborative work, we determine also the field of maturing intellectual functions; functions in imminent stage of development should bear fruits and consequently, be transferred to the child's real mental level of development. (Vygotski, 2004 in Prestes, 2012, p. 206).

Collective and shared analysis with Grucomat of material produced with the students, has enabled us to understand the senses and meanings that they attribute to different words and uses of probabilistic language, in this case for a group of students from families of low economic power, whose experiences should be taken into account. In this sense, coexistence with those students allowed the researchers to comprehend the different dimensions of their lifestyles, and the learning opportunities that they generate for developing scientific knowledge. Spontaneous concepts brought from those experiences must become questions and be related to scientific concepts, enabling new ways of life and statistical literacy, as supported by Watson (2006).

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